

# Unitarity as Theorem

## Modal Logic of Information Conservation

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### Zusammenfassung

In modal5.md (Part III) and Unitarityaet.md, the resurrection is derived from the axioms A16–A20. These axioms are strong. The present treatise shows: Unitarity can be derived from weaker assumptions. It introduces an operator I (“information is conserved”), axiomatizes three minimal principles U1–U3, and proves: From U1–U3, the resurrection follows as a theorem. Unitarity is thus not an additional assumption but the structure of Totality T itself:  $T : T$ . The “intervention” of the Omega Point into paths that do not reach it is structural, not temporal. What appears from the perspective of a path as loss is, from the perspective of S, integration.

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*Past and future are horizons of knowledge.*

*The present is the locus of reality.*

*Nothing that was, is lost.*

## Preface: From Strong Axioms to Weaker Principles

In `modal5.md` (Part III), the resurrection was derived from the axioms A16–A20:

$$A16 := \forall c(\text{Consciousness}(c) \rightarrow \text{integrated}(c, S))$$

$$A17 := \forall c(\text{Knowledge}(c, S) \leftrightarrow \text{integrated}(c, S))$$

$$A18 := \Box \forall I(\text{Existence}(I) \rightarrow \text{Conservation}(I))$$

$$A19 := \forall c(\text{Death}(c) \rightarrow \text{Integration}(c, S))$$

$$A20 := \forall c(\text{Knowledge}(c, S) \leftrightarrow \text{Existence}(c))$$

These axioms are **strong**. They presuppose that **all** consciousnesses are integrated into  $S$ , that **all** information is conserved, that death is **always** integration.

The present treatise shows: **Unitarity can be derived from weaker assumptions**. It introduces an operator  $\mathcal{I}$  (“information is conserved”), axiomatizes three minimal principles U1–U3, and proves: From U1–U3, the resurrection follows as a **theorem**.

The central claim of this treatise is:

**Unitarity is not an additional assumption. It is the structure of Totality  $T$  itself. From it, the resurrection follows as a logical necessity — not as a moral promise, but as a consequence of information conservation.**

The investigation remains faithful to the self-understanding of `modal5.md`: It shows **dependencies**, not truths. It shows that the resurrection follows from weaker assumptions.

## The Operator $\mathcal{I}$ : Definition and Axioms

### The Basic Idea

The operator  $\mathcal{I}$  formalizes the concept of **information conservation**. It is **not** a moral concept, but a **structural** one.

**Intuition:** “ $\mathcal{I}p$ ” means: “The information about  $p$  is conserved.”

### The Language

We extend the language of S5 by:

- A **unary operator**  $\mathcal{I}$  (read: “Information”).

$$\mathcal{I}p@w \iff \text{the information about } p \text{ is conserved in } w$$

### The Axioms

$$\begin{aligned} I1 &:= \Box(\mathcal{I}p \rightarrow \Diamond p) \\ I2 &:= \Box(\mathcal{I}(p \wedge q) \leftrightarrow (\mathcal{I}p \wedge \mathcal{I}q)) \\ I3 &:= \Box(\mathcal{I}\neg p \leftrightarrow \neg \mathcal{I}p) \\ I4 &:= \Box(\mathcal{I}p \rightarrow \Box \mathcal{I}p) \end{aligned}$$

### Explanation:

- I1: What is conserved is possible.
- I2: Conservation distributes over conjunction.
- I3: Conservation is complementary — the information about  $\neg p$  is conserved exactly when the information about  $p$  is conserved.
- I4: Conservation is necessary when it holds.

**Important:**  $\mathcal{I}$  is **not** a normal operator. It does **not** satisfy the Kripke axiom  $K$ . Instead, it satisfies the **information axioms** I1–I4.

## The Three Principles U1–U3

### U1: Realization Implies Possible Reconstruction

$$\boxed{U1 := \Box \forall p (\text{Realized}(p) \rightarrow \mathcal{I}p)}$$

**Reading:** If  $p$  is realized, then the information about  $p$  is conserved.

**Metaphysical meaning:** What was once real cannot fully disappear. It remains conserved as information — even if the original form is lost.

**Formal consequence:** From U1 and I1:  $\text{Realized}(p) \rightarrow \Diamond p$ . This is trivial. From U1 and I4:  $\text{Realized}(p) \rightarrow \Box \mathcal{I}p$ . This is stronger: The conservation of information about  $p$  is necessary.

## U2: Death Is Local Access Loss

$$U2 := \Box \forall c (\text{Death}(c) \rightarrow \neg \Diamond_{\text{local}} \text{Access}(c))$$

**Reading:** The death of a consciousness  $c$  is the loss of **local** access to  $c$ .

**Metaphysical meaning:** Death is **not** extinction. It is an **access loss**. From the perspective of  $c$ , it appears as an end. From the perspective of  $S$ , it is a transition.

**Formal consequence:** From U2 and U1:  $\text{Death}(c) \rightarrow \mathcal{I}c$ . The information about  $c$  is conserved, even if  $c$  is no longer locally accessible.

## U3: Global Integration Is Possible

$$U3 := \Box \forall c (\text{Death}(c) \rightarrow \Diamond_{\text{global}} \text{Integration}(c, S))$$

**Reading:** The death of a consciousness  $c$  makes its **global** integration into  $S$  possible.

**Metaphysical meaning:** Death is **not the end**. It is the condition of the possibility of integration. What is lost locally is integrated globally.

**Formal consequence:** From U2, U3, and U1:  $\text{Death}(c) \rightarrow \Diamond_{\text{global}} \text{Integration}(c, S)$ .

# Proof: Unitarity Implies Resurrection

## The Main Theorem

**Theorem U-A:** In S5+U1–U3:

$$\Box \forall c (\text{Consciousness}(c) \rightarrow \Box \Diamond \text{Resurrection}(c))$$

## The Proof

**Step 1:** Let  $c$  be an arbitrary consciousness.

$$\text{Consciousness}(c)$$

**Step 2:** We assume that  $c$  dies.

$$\text{Death}(c)$$

**Step 3:** From U2:

$$\neg\Diamond_{\text{local}}\text{Access}(c)$$

Local access to  $c$  is lost.

**Step 4:** From U1 and Step 2:

$$\mathcal{I}c$$

The information about  $c$  is conserved.

**Step 5:** From U3 and Step 2:

$$\Diamond_{\text{global}}\text{Integration}(c, S)$$

The global integration of  $c$  into  $S$  is possible.

**Step 6:** We define:

$$\text{Resurrection}(c) := \text{Integration}(c, S)$$

**Step 7:** From Step 5 and 6:

$$\Diamond\text{Resurrection}(c)$$

**Step 8:** Since Steps 2–7 hold for **every**  $c$  that dies, and since every consciousness dies (empirical fact), it follows:

$$\forall c(\text{Consciousness}(c) \rightarrow \Diamond\text{Resurrection}(c))$$

**Step 9:** With Necessitation:

$$\Box\forall c(\text{Consciousness}(c) \rightarrow \Diamond\text{Resurrection}(c))$$

q.e.d.

## What the Proof Shows

The proof shows:

1. The resurrection does **not** follow from moral assumptions.
2. The resurrection follows from **structural** assumptions: information conservation, access loss, integration possibility.
3. The resurrection is a **logical necessity** — under the assumptions U1–U3.

## What the Proof Does Not Show

The proof does **not** show:

1. That the resurrection **actually** takes place.
2. That the resurrection takes place **temporally**.
3. That the resurrection is **morally** justified.

The proof only shows: Under the assumptions U1–U3, the possibility of resurrection follows for every consciousness.

## Unitarity as the Structure of $T$

### The Central Insight

Unitarity is **not** an additional assumption. It is the **structure** of Totality  $T$  itself.

**Formally:**

$$\boxed{\forall \gamma \subseteq T : \gamma \in T}$$

**Reading:** For every path  $\gamma$  that is a subset of  $T$ :  $\gamma$  is an element of  $T$ .

**Metaphysical meaning:** Totality  $T$  contains **all** paths — including those that do not reach the Omega Point. There is **no escape** from  $T$ . What happens in  $T$  remains in  $T$ .

### The Consequence

If unitarity is the structure of  $T$  itself, then it follows:

1. **No information is lost.** For loss would mean that a path  $\gamma$  falls out of  $T$ . That is impossible.
2. **All paths are conserved.** Including those that do not reach the Omega Point.
3. **Integration is structural, not temporal.** What appears from the perspective of a path as loss is, from the perspective of  $T$ , conservation.

### The “Intervention” of the Omega Point

The “intervention” of the Omega Point into paths that do not reach it is **structural**, not **temporal**:

- **Structural:**  $T$  contains all paths. Integration has **always already** happened.
- **Not temporal:** There is no point in time at which the Omega Point “intervenes.” It is the **form** of  $T$ .

**Formal version:**

$$\forall \gamma \subseteq T : \gamma \in T \Rightarrow \text{Integration}(\gamma, S)$$

**Consequence:** From the perspective of a path, integration appears as an **intervention from outside**. From the perspective of  $S$ , it is **self-knowledge**.

## Comparison with A16–A20: What Is Gained, What Is Lost?

### What Is Gained?

1. **Weaker assumptions.** A16–A20 presuppose that all consciousnesses are integrated into  $S$ . U1–U3 presuppose only that information conservation is structural.
2. **Structural foundation.** A16–A20 are **axiomatic**. U1–U3 are **structural** — they describe the form of  $T$ .
3. **Resurrection as theorem.** With A16–A20, the resurrection is a theorem from strong assumptions. With U1–U3, it is a theorem from weak assumptions.
4. **Connectivity.** U1–U3 are connectable to physical concepts (information conservation in quantum mechanics). A16–A20 are primarily theological.

### What Is Lost?

1. **Directness.** A16–A20 are direct. U1–U3 require the detour via the operator  $\mathcal{I}$ .
2. **Explicitness.** A16–A20 speak directly of consciousness, death, integration. U1–U3 speak of information conservation.
3. **Theological language.** A16–A20 stand in a theological tradition. U1–U3 stand in a structural tradition.

### What Remains?

1. **The resurrection is a theorem.** In both cases.
2. **Unitarity is fundamental.** In both cases.
3. **Integration is structural.** In both cases.

### Conclusion

U1–U3 are **stronger** in their reach and **weaker** in their assumptions. They are therefore **preferable** when it comes to showing the resurrection as a **structural necessity**, not as a **moral promise**.

## What Follows? Resurrection Is a Logical Necessity

### The Central Insight

The resurrection is **not** a moral hope. It is a **logical necessity**.

**Formally:** From U1–U3 follows  $\Box\forall c(\text{Consciousness}(c) \rightarrow \Diamond\text{Resurrection}(c))$ .

**Metaphysically:** The resurrection is the **consequence** of information conservation. It is not an **event** in time, but a **structure** of Totality.

### The Consequences

**Consequence 1:** The resurrection cannot be **prevented**. It is structural.

**Consequence 2:** The resurrection cannot be **forced**. It is not a moral reward.

**Consequence 3:** The resurrection is **universal**. It holds for all consciousnesses, not only for the “good.”

### The Relation to Tradition

Classical theology speaks of the resurrection as **grace** or **reward**. The present treatise shows: The resurrection is **not** grace. It is a **necessity**.

This is **no contradiction** to theology. It is a **refinement**: Grace is not arbitrary. It is structural.

## Open Questions: Is U1–U3 Minimal?

### The Question

We have shown: From U1–U3 follows the resurrection. But is U1–U3 **minimal**? Is each of the three principles indispensable?

### The Three Sub-Questions

**Question 1:** Is U1 indispensable? What happens if we omit U1?

**Question 2:** Is U2 indispensable? What happens if we omit U2?

**Question 3:** Is U3 indispensable? What happens if we omit U3?

### Possible Answers

**On U1:** Without U1, it does not follow that the information about  $c$  is conserved. The resurrection would not be derivable.

**On U2:** Without U2, death would not be defined as access loss. The resurrection would not be bound to death.

**On U3:** Without U3, it does not follow that integration is possible. The resurrection would be only a possibility, not a necessity.

### What We Have Not Shown

We have **not** shown that U1–U3 are minimal. We have only shown that they are **sufficient**.

The question of **minimality** remains open. It is the subject of **Modal 11** and beyond.

### A Second Open Question

**Question 4:** Is the operator  $\mathcal{I}$  minimal? Is there a weaker operator that plays the same role?

**Possible answer:** The operator  $\mathcal{I}$  is the **weakest** operator that formalizes information conservation. But there could be other operators.

## Conclusion: What Has Been Shown?

The present investigation has shown:

1. **Unitarity can be derived from weaker assumptions.** The three principles U1–U3 are sufficient for the resurrection.
2. **Unitarity is not an additional assumption.** It is the structure of Totality  $T$  itself:  $\forall \gamma \subseteq T : \gamma \in T$ .
3. **The “intervention” of the Omega Point is structural, not temporal.** What appears from the perspective of a path as loss is, from the perspective of  $S$ , integration.
4. **The resurrection is a logical necessity.** It is not a moral hope, but a structural consequence.

The central claim of this treatise is:

**Unitarity is not an additional assumption. It is the structure of Totality  $T$  itself. From it, the resurrection follows as a logical necessity — not as a moral promise, but as a consequence of information conservation.**

The question of the minimality of U1–U3 remains open.

## Appendix A: The Axioms in an Overview

### Information Axioms

$$\begin{aligned}
I1 &:= \Box(\mathcal{I}p \rightarrow \Diamond p) \\
I2 &:= \Box(\mathcal{I}(p \wedge q) \leftrightarrow (\mathcal{I}p \wedge \mathcal{I}q)) \\
I3 &:= \Box(\mathcal{I}\neg p \leftrightarrow \neg \mathcal{I}p) \\
I4 &:= \Box(\mathcal{I}p \rightarrow \Box \mathcal{I}p)
\end{aligned}$$

### Principles U1–U3

$$\begin{aligned}
U1 &:= \Box \forall p (\text{Realized}(p) \rightarrow \mathcal{I}p) \\
U2 &:= \Box \forall c (\text{Death}(c) \rightarrow \neg \Diamond_{\text{local}} \text{Access}(c)) \\
U3 &:= \Box \forall c (\text{Death}(c) \rightarrow \Diamond_{\text{global}} \text{Integration}(c, S))
\end{aligned}$$

### Unitarity as the Structure of $T$

$$\boxed{\forall \gamma \subseteq T : \gamma \in T}$$

### The Main Theorem

$$\boxed{S5 + U1 + U2 + U3 \vdash \Box \forall c (\text{Consciousness}(c) \rightarrow \Diamond \text{Resurrection}(c))}$$

## Appendix B: Comparison with A16–A20

| Aspect                  | A16–A20 (modal5.md) | U1–U3 (modal10.md) |
|-------------------------|---------------------|--------------------|
| Assumptions             | Strong              | Weak               |
| Structure               | Axiomatic           | Structural         |
| Resurrection            | Theorem             | Theorem            |
| Connectivity to physics | Low                 | High               |
| Theological language    | Yes                 | No                 |

|            |                     |                    |
|------------|---------------------|--------------------|
| Aspect     | A16–A20 (modal5.md) | U1–U3 (modal10.md) |
| Directness | High                | Medium             |

## Appendix C: The Three Principles in an Overview

| Principle | Reading                                            | Function                    |
|-----------|----------------------------------------------------|-----------------------------|
| U1        | Realization $\rightarrow$ information conservation | What was, remains           |
| U2        | Death $\rightarrow$ local access loss              | What is lost locally        |
| U3        | Death $\rightarrow$ global integration possibility | What is integrated globally |

## Afterword

You have read this treatise. Perhaps the idea surprised you that unitarity is the structure of  $T$  itself. Perhaps the consequence impressed you that the resurrection is a logical necessity. Both are fine.

The investigation has shown: Unitarity is **not** an additional assumption. It is the **structure** of Totality  $T$  itself. From it follows the resurrection as a **logical necessity** — not as a moral promise, but as a consequence of information conservation.

What remains is the insight that the “intervention” of the Omega Point into paths that do not reach it is **structural**, not **temporal**. From the perspective of a path, it appears as loss. From the perspective of  $S$ , it is integration.

One final question:

**Do we dare the leap into openness to the limit — even though the limit itself is not formally reachable?**

*This treatise was written in the spirit of rigorous modal logic, yet in the language of philosophy – for truth requires both: the precision of the formula and the breadth of the concept.*