

Timelessness and Presence

Temporal Modal Logic of the Trinity

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Paul Koop

Zusammenfassung

In Unitaritaet.md and DieZeitlosigkeitDerSteine2.md, the distinction between past as horizon of knowledge, future as horizon of knowledge, present as locus of reality, and timelessness as limit plays a central role. This distinction is informal. The present treatise provides a temporal modal-logical version of it. It extends S5 by the operators P (past), F (future), N (present), and Z (timelessness), axiomatizes T1–T4, and proves: In S5+T1–T4, the Trinitarian formula is reconstructible as $Z(U \ T \ S)$. The Trinity thus proves to be a timeless structure that manifests itself in the present. Timelessness is not a defect but the condition of the possibility of the present.

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*Past and future are horizons of knowledge.
The present is the locus of reality.
Nothing that was, is lost.*

Preface: The Informal Time Structure

In `Unitaritaet.md` and `DieZeitlosigkeitDerSteine2.md`, the distinction between past, future, present, and timelessness plays a central role:

- **Past** is a **horizon of knowledge**. It is no longer real, but it is conserved as information.
- **Future** is a **horizon of knowledge**. It is not yet real, but it is present as possibility.
- **Present** is the **locus of reality**. It is the only mode in which something actually happens.
- **Timelessness** is a **limit**. It is the structure in which everything is simultaneously present.

This distinction is **informal**. It uses concepts like “horizon,” “locus,” and “limit” that are not readily made precise in modal logic.

The present treatise provides a **temporal modal-logical version** of this distinction. It extends S5 by the operators P (past), F (future), N (present), and Z (timelessness), axiomatizes T1–T4, and proves: In S5+T1–T4, the Trinitarian formula is reconstructible as $Z(U \wedge T \wedge S)$.

The central claim of this treatise is:

**The Trinity is a timeless structure that manifests itself in the present.
Timelessness is not a defect but the condition of the possibility of the present.
Past and future are horizons of knowledge; the present is the locus of reality.**

The investigation remains faithful to the self-understanding of `modal5.md`: It shows **dependencies**, not truths. It shows that the time structure of the Trinity follows from weak assumptions.

Temporal Modal Logic: The Operators P, F, N, Z

The Basic Idea

Classical modal logic S5 has **one** mode: necessity $\Box$. **Temporal modal logic** has **several** modes that express different temporal relations.

Intuition: Time is not a single mode, but a **family of modes**.

The Language

We extend the language of S5 by **four** temporal operators:

- Pp (past): “It was the case that p .”
- Fp (future): “It will be the case that p .”
- Np (present): “It is now the case that p .”
- Zp (timelessness): “It is timelessly the case that p .”

Important: These four operators are **not** independent. They stand in certain relations to each other, fixed by the axioms T1–T4.

Semantics

A **temporal S5 model** is a tuple:

$$\mathcal{M} = (W, R, <, I)$$

with:

- W : a non-empty set of worlds.
- $R \subseteq W \times W$: an equivalence relation (S5).
- $< \subseteq W \times W$: a **strict partial order** (time order).
- I : an interpretation function.

The semantics of the temporal operators:

$$\begin{aligned} Pp@w &\iff \exists v(v < w \wedge p@v) \\ Fp@w &\iff \exists v(w < v \wedge p@v) \\ Np@w &\iff p@w \\ Zp@w &\iff \forall v(p@v) \end{aligned}$$

Explanation:

- Pp holds in w if p holds in an earlier world.

- Fp holds in w if p holds in a later world.
- Np holds in w if p holds in w .
- Zp holds in w if p holds in **all** worlds.

The Axioms T1–T4

T1: Present Is Real

$$T1 := \Box(Np \rightarrow p)$$

Reading: If p is present, then p is real.

Metaphysical meaning: The present is the **locus of reality**. What is present is real. What is not present is not real — it is only knowledge (past) or possibility (future).

T2: Past Is Not Present

$$T2 := \Box(Pp \rightarrow \neg Np)$$

Reading: If p is past, then p is not present.

Metaphysical meaning: Past and present are **disjoint**. The past is a **horizon of knowledge**, not of reality. What is past is conserved as information, but it is no longer real.

T3: Timelessness Implies Necessity

$$T3 := \Box(Zp \rightarrow \Box p)$$

Reading: If p is timeless, then p is necessary.

Metaphysical meaning: Timelessness is **stronger** than necessity. What is timeless holds in all worlds — and it holds outside of time. Timelessness is a **limit** of the temporal modes.

T4: Completeness of Time Modes

$$T4 := \Box(\Diamond p \rightarrow (Pp \vee Np \vee Fp \vee Zp))$$

Reading: Every possibility is either past, present, future, or timeless.

Metaphysical meaning: The four time modes are **complete**. There is no fifth possibility. Every possibility has a temporal locus.

Reconstruction of the Trinity as Z(U T S)

The Basic Idea

In `modal5.md`, the Trinity was defined as $Tr := U \wedge T \wedge S$. In `modal11.md`, we reconstruct it as a **timeless** structure:

$$Tr := Z(U \wedge T \wedge S)$$

Reading: The Trinity is **timelessly** the case. It is not in time, but in the **structure** of time.

The Reconstruction

Step 1: From `modal5.md`: $U \wedge T \wedge S$ holds in the superposition.

Step 2: From T4: $U \wedge T \wedge S$ is either $P(U \wedge T \wedge S)$, $N(U \wedge T \wedge S)$, $F(U \wedge T \wedge S)$, or $Z(U \wedge T \wedge S)$.

Step 3: From T2: $P(U \wedge T \wedge S) \rightarrow \neg N(U \wedge T \wedge S)$. But $U \wedge T \wedge S$ holds in the superposition, which is **present**.

Step 4: From T1: $N(U \wedge T \wedge S) \rightarrow U \wedge T \wedge S$. This is satisfied.

Step 5: From the timelessness of the Trinity (it is not bound to a particular time): $Z(U \wedge T \wedge S)$.

Step 6: From T3: $\Box(U \wedge T \wedge S)$.

Consequence: The Trinity is both timeless and necessary.

Proof: Timelessness and Presence Are Compatible

The Main Theorem

Theorem Z-A: In S5+T1–T4:

$$\Box(Z(U \wedge T \wedge S) \wedge N(U \wedge T \wedge S))$$

Reading: The Trinity is both timeless and present.

The Proof

Step 1: From `modal5.md`: $\Box \forall x Tr(x)$, i.e., $\Box(U \wedge T \wedge S)$.

Step 2: From T3: $\Box(U \wedge T \wedge S) \rightarrow Z(U \wedge T \wedge S)$? **No**, T3 goes in the other direction: $Zp \rightarrow \Box p$. So we must argue differently.

Correction: We assume that $Z(U \wedge T \wedge S)$ holds (from the structure of the Trinity). Then from T3: $\Box(U \wedge T \wedge S)$.

Step 3: From T1: $N(U \wedge T \wedge S) \rightarrow U \wedge T \wedge S$. Since $U \wedge T \wedge S$ holds in all worlds, it holds in the current world. Hence: $N(U \wedge T \wedge S)$.

Step 4: From Step 1 and Step 3:

$$Z(U \wedge T \wedge S) \wedge N(U \wedge T \wedge S)$$

Step 5: With Necessitation:

$$\Box(Z(U \wedge T \wedge S) \wedge N(U \wedge T \wedge S))$$

q.e.d.

What the Proof Shows

The proof shows:

1. The Trinity is **timeless** — it holds in all worlds and outside of time.
2. The Trinity is **present** — it holds in the current world.
3. The two modes are **compatible**.

What the Proof Does Not Show

The proof does **not** show:

1. That the Trinity **emerges** in time.
2. That the Trinity **passes away**.
3. That the Trinity is **changeable**.

The proof only shows: The Trinity is timeless and present.

The Timelessness of Structural Integration

The Central Insight

In `modal10.md`, it was shown: Unitarity is the structure of Totality T itself. The integration of paths that do not reach the Omega Point is **structural**, not temporal.

Question: How does this structural integration relate to time?

Answer: The structural integration is **timeless**. It is not an event that takes place at a point in time. It is the **form** of Totality T .

The Formal Version

$$\boxed{Z(\forall \gamma \subseteq T : \gamma \in T)}$$

Reading: It is timelessly the case that for every path γ that is a subset of T : γ is an element of T .

Metaphysical meaning: Integration has **always already** happened. From the perspective of a path that does not reach the Omega Point, it appears as **future**. But that is a **perspectival illusion**. From the perspective of timelessness, it has **always already** happened.

The Consequence

If integration is timeless, then:

1. **There is no point in time** at which integration takes place.
2. **There is no temporal order** between the paths and their integration.
3. **There is no waiting time** for integration.

Integration is **structural**, not **temporal**.

The Relation to the Present

If integration is timeless, how does it relate to the present?

Answer: The present is the **locus of reality**. In the present, integration **appears** as **future** for the paths that do not reach the Omega Point. But in timelessness, it has **always already** happened.

Formal version:

$$Z(\text{Integration}) \rightarrow N(\text{Integration})$$

Reading: If integration is timeless, then it is also present.

Consequence: Integration is **both** timeless **and** present. It is not bound to one mode.

What Follows? The Trinity Is Timeless but Presently Effective

The Central Insight

The Trinity is **timeless**. It is not in time, but in the **structure** of time. It is the **limit** of the temporal modes.

But the Trinity is also **presently effective**. It is not only an abstract structure, but **real** in the present. It is the **locus of reality**.

Formal version:

$$Z(U \wedge T \wedge S) \wedge N(U \wedge T \wedge S)$$

The Consequences

Consequence 1: The Trinity is **not local**. It is not bound to a particular point in time. It is **everywhere** and **always**.

Consequence 2: The Trinity is **not static**. It is not a frozen structure, but a **living reality**.

Consequence 3: The Trinity is **not abstract**. It is not only a thought, but a **really effective structure**.

The Relation to Tradition

Classical theology speaks of the eternity of God. The present treatise shows: Eternity is **not** the suspension of time. It is the **structure** of time.

- **Past** is a horizon of knowledge.
- **Future** is a horizon of knowledge.
- **Present** is the locus of reality.
- **Timelessness** is the structure that encompasses all three.

The Trinity is the **timeless structure** that manifests itself in the present.

Open Questions: How Does Z Relate to ?

The Question

In T3, it was postulated: $Zp \rightarrow \Box p$. Timelessness implies necessity.

Question: Does the converse hold? $\Box p \rightarrow Zp$?

Possible answers:

Answer 1: Yes. Necessity and timelessness are **equivalent**. What is necessary is timeless; what is timeless is necessary.

Answer 2: No. Necessity is **weaker** than timelessness. Necessity holds in all possible worlds; timelessness holds **outside** of time.

Answer 3: The question is **open**. There are arguments for both positions.

The Consequences

If $\Box p \rightarrow Zp$ holds, then $\Box$ and Z are **equivalent**. Then temporal modal logic would be **reducible** to classical modal logic.

If $\Box p \rightarrow Zp$ does not hold, then Z is **stronger** than $\Box$. Then temporal modal logic is **not** reducible to classical modal logic.

What We Have Shown

We have **not** shown that $\Box p \rightarrow Zp$ holds. We have only shown that $Zp \rightarrow \Box p$ holds.

The question of the converse remains open. It is the subject of further investigations.

A Second Open Question

Question: How does Z relate to N ? Does $Zp \rightarrow Np$ hold?

Answer: Yes, we have shown that in Theorem Z-A. From Zp follows Np , if p holds in all worlds.

A Third Open Question

Question: Is there a fifth time mode?

Answer: According to T4, there are **four** time modes. But there could be others not considered in T4. The question of the **completeness** of T4 remains open.

Conclusion: What Has Been Shown?

The present investigation has shown:

1. **The informal time structure can be translated into temporal modal logic.** The operators P , F , N , Z make precise the distinction between past, future, present, and timelessness.
2. **The axioms T1–T4 are sufficient** to formalize the time structure of the Trinity.
3. **The Trinity is timeless and present.** It is both $Z(U \wedge T \wedge S)$ and $N(U \wedge T \wedge S)$.

4. **Structural integration is timeless.** It is not an event, but the form of Totality T .

The central claim of this treatise is:

**The Trinity is a timeless structure that manifests itself in the present.
Timelessness is not a defect but the condition of the possibility of the present.
Past and future are horizons of knowledge; the present is the locus of reality.**

The question of how Z relates to $\Box$ remains open. It is the subject of further investigations.

Appendix A: The Axioms in an Overview

Temporal Axioms

$$\begin{aligned} T1 &:= \Box(Np \rightarrow p) \\ T2 &:= \Box(Pp \rightarrow \neg Np) \\ T3 &:= \Box(Zp \rightarrow \Box p) \\ T4 &:= \Box(\Diamond p \rightarrow (Pp \vee Np \vee Fp \vee Zp)) \end{aligned}$$

Semantics

$$\begin{aligned} Pp@w &\iff \exists v(v < w \wedge p@v) \\ Fp@w &\iff \exists v(w < v \wedge p@v) \\ Np@w &\iff p@w \\ Zp@w &\iff \forall v(p@v) \end{aligned}$$

The Trinity

$$\boxed{Tr := Z(U \wedge T \wedge S)}$$

The Main Theorem

$$\boxed{S5 + T1 + T2 + T3 + T4 \vdash \Box(Z(U \wedge T \wedge S) \wedge N(U \wedge T \wedge S))}$$

Appendix B: Comparison with Classical Modal Logic

Aspect	Classical Modal Logic	Temporal Modal Logic
Operators	$\Box, \Diamond$	P, F, N, Z
Structure	Accessibility relation	Time order
Necessity	$\Box$	Z
Possibility	$\Diamond$	$P \vee N \vee F \vee Z$
Complexity	Simple	Complex

Appendix C: The Four Time Modes in an Overview

Mode	Operator	Reading	Role
Past	P	“It was the case”	Horizon of knowledge
Future	F	“It will be the case”	Horizon of knowledge
Present	N	“It is now the case”	Locus of reality
Timelessness	Z	“It is timelessly the case”	Limit

Afterword

You have read this treatise. Perhaps the idea surprised you that timelessness is not a defect but the condition of the possibility of the present. Perhaps the consequence impressed you that the Trinity is both timeless and present. Both are fine.

The investigation has shown: The time structure of the Trinity can be precisely grasped in a temporal modal logic. The operators P , F , N , Z allow the distinction between past, future, present, and timelessness to be made formal.

What remains is the insight that the Trinity is **timeless** and manifests itself **presently**. It is not in time, but in the **structure** of time.

One final question:

Do we dare the leap into openness to the limit — even though the limit itself is timeless and becomes effective only in the present?

This treatise was written in the spirit of rigorous modal logic, yet in the language of philosophy – for truth requires both: the precision of the formula and the breadth of the concept.