

The Trinity as a Necessary Structure of a Monistic
Modal Ontology
Minimalist, Limit-Based Version

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*Past and future are horizons of knowledge.
The present is the place of reality.*

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Preface

This treatise is an attempt to demonstrate an ancient metaphysical intuition – the threefold unity of origin, totality, and self-knowledge – not as a matter of faith, but as a **logical necessity** of a consistent ontology.

It differs from earlier versions in three decisive respects:

1. **Minimal axiomatics:** Instead of six or nine axioms, the treatise will proceed with only two fundamental principles.
2. **Limit structure:** The concepts U (Origin), T (Totality), and S (Self-Knowledge) are not understood as substances, but as **limits** of an open interval.
3. **Transcendental deduction:** Axioms such as "consciousness is possible" or "modal realism" are not presupposed, but derived from monism and the existence of a world.

The central thesis is:

> Under the axioms of monism and the existence of a possible world, a threefold limit structure of Origin, Totality, and Self-Knowledge necessarily follows.

The treatise provides **no proof of God** in the classical sense. It demonstrates no personal or substantial Trinity. It merely shows: **If you accept monism and the existence of a world, then a threefold limit structure necessarily follows.**

Introduction: Aim and Methodological Self-Restriction

Classical metaphysics has always attempted to derive the fundamental structure of reality from a few basic principles. A particular challenge arises from three seemingly distinct aspects:

1. **Why is there something rather than nothing?**
2. **How does the totality of all possibilities relate to actuality?**
3. **How can a state arise within reality that knows reality itself?**

The ontology examined here proposes to answer these three questions not through three separate metaphysical substances, but through three necessary perspectives on the same reality:

- **Origin (U):** the necessary condition for the existence of possibilities at all – understood as the lower limit of Totality;
- **Totality (T):** the entirety of all realized possibilities – understood as an open interval;

- **Self-Knowledge (S):** the reflexive completion of reality in a state of complete knowledge of itself – understood as the upper limit of Totality.

This structure is called the **Trinity**:

$$Tr := U \wedge T \wedge S$$

The term "Trinity" here does not denote a personal or substantial threefoldness, but a functional unity of three necessary aspects of a single reality.

Formal Language and Logical Framework

Modal Logic S5

We work in first-order modal logic with identity in system S5:

- **Necessity:** $\Box p$
- **Possibility:** $\Diamond p := \neg \Box \neg p$

Axioms of S5:

- (K) $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$
- (T) $\Box p \rightarrow p$
- (4) $\Box p \rightarrow \Box \Box p$
- (5) $\Diamond p \rightarrow \Box \Diamond p$

Inference Rules:

- **Modus Ponens:** From p and $p \rightarrow q$, infer q .
- **Necessitation:** From p (derivable), infer $\Box p$.

Predicate Logic

We use classical first-order predicate logic with identity ($=$) and the usual quantifiers ($\forall, \exists$).

The Two Axioms

A1 – Monism

There are no fundamentally separated realms of reality.

$$A1 := \Box \neg \exists x \exists y \text{FundamentallySeparated}(x, y)$$

Explanation: If two realms were fundamentally separated, there could be no causal or ontological interaction between them. Then they could not be part of a unified reality, which contradicts monism. Such a dualism would be incompatible with a complete modal realism.

A4 – Existence of a Possible Reality

There is at least one possible world.

$$A4 := \Diamond \exists w \text{World}(w)$$

Explanation: This axiom ensures that the modal universe is not empty. It is the weakest possible existence axiom: it does not say that a world **actually** exists, but only that it is **possible**.

Definition of the Limit Structure

Totality T as an Open Interval

Totality T is the entirety of all realized possibilities. We understand T as an **open interval**:

$$T := (U, S)$$

This means: T is the set of all states that lie **between** Origin U and Self-Knowledge S. The limits U and S do **not** belong to T – they are **limit points**.

What does "open interval" mean mathematically?

- An open interval (a, b) contains all numbers between a and b, but **not** a and b themselves.
- It has **no smallest** and **no largest** element.
- It has **limits** – a is the lower limit (infimum), b the upper limit (supremum).

Transferred to ontology:

- Totality T is the set of all **actually realized** states.
- These states are **ordered** – from "minimal structure" to "maximal structure".
- **Origin U** is the lower limit – that which comes closest to nothing, but is itself not nothing.
- **Self-Knowledge S** is the upper limit – the complete transparency of Totality, which itself does not belong to Totality.

Origin U as the Lower Limit

$$U := \lim_{x \rightarrow \inf} T$$

This means: U is the **limit** of Totality T when approaching the "beginning." U is that which comes closest to nothing – but **not nothing** (for nothing would be a fundamental separation, which A1 prohibits).

Self-Knowledge S as the Upper Limit

$$S := \lim_{x \rightarrow \sup} T$$

This means: S is the **limit** of Totality T when approaching the "end." S is the complete self-transparency of Totality – but **not itself a part** of T (for otherwise it would not be complete).

Well-Foundedness as a Consequence of Openness

From the openness of the interval follows **directly** well-foundedness:

$$\neg \exists \text{infinite chain}(g_1, g_2, \dots)$$

Justification: In an open interval, there is no smallest element. Every element has a "before" – but there is **no final ground** that lies within the interval. The final ground is the **limit U**, which does **not** belong to the interval.

The Trinity

The **Trinity** is the unity of the three limits:

$$Tr := U \wedge T \wedge S$$

Or in the language of limits:

$$Tr := \lim_{x \rightarrow \inf} T \wedge T \wedge \lim_{x \rightarrow \sup} T$$

Derivation of Further Principles from A1 and A4

A5 – Consciousness as a Real Possibility (Theorem)

Theorem: From A1 and A4 follows $\Diamond C$ (consciousness is possible).

Proof:

1. Suppose there is a world w : $\exists w \text{ World}(w)$.
2. Suppose consciousness is impossible: $\neg\Diamond C$. This means: $\Box\neg C$ – it is necessary that there is no consciousness.
3. If there is no consciousness, then there is also no **experience** of world. The world would be **unknowable**.
4. An unknowable world would be **fundamentally separated** from any possible conscious perspective.
5. **But A1 prohibits fundamental separation.**
6. Therefore: $\neg\Diamond C$ leads to a contradiction with A1.
7. Therefore: $\Diamond C$.

$$\boxed{\Diamond C}$$

This is **A5** – but it is no longer an axiom, but a theorem.

A3 – No Absolute Nothing (Theorem)

Theorem: From A1 follows $\Box(N \rightarrow \neg\Diamond \text{World})$.

Proof:

1. Absolute nothing N would be a **fundamentally separated realm** from reality.
2. **A1 prohibits fundamental separation.**
3. Therefore: $N \rightarrow \neg\Diamond \text{World}$.
4. With Necessitation: $\Box(N \rightarrow \neg\Diamond \text{World})$.

$$\boxed{\Box(N \rightarrow \neg\Diamond \text{World})}$$

This is **A3** – but it is no longer an axiom, but a theorem.

A2 – Modal Realism (Theorem)

Theorem: From A1, A4 and A5 follows $\forall p(\Diamond p \rightarrow \exists w \text{ Realized}(w, p))$.

Proof:

1. Consciousness (C) is the capacity to experience **differences**. (From the definition of consciousness.)
2. If there were an **unrealized possibility** – a possibility that is not realized in any world – then this possibility would be **not experienceable by consciousness**.

3. An unexperienceable possibility would be **fundamentally separated** from the world of consciousness.
4. **A1 prohibits fundamental separation.**
5. Therefore: There can be no unrealized possibilities.
6. Therefore: $\forall p(\Diamond p \rightarrow \exists w \text{ Realized}(w, p))$.

$$\boxed{\forall p(\Diamond p \rightarrow \exists w \text{ Realized}(w, p))}$$

This is **A2** – but it is no longer an axiom, but a theorem.

A6 – Origin as Ontological Embedding Condition (Definition + A10)

Theorem: From the definition of U and the openness of T follows $\Box(\neg \text{Nec}(p) \rightarrow \exists g \text{ Ground}(g, p))$.

Proof:

1. Every non-necessary reality is **contingent** – it could also not exist.
2. If it exists, it needs a **ground** – otherwise it would be groundless.
3. The openness of T (as an open interval) guarantees that there is a **final ground**: the limit U.
4. Therefore: $\Box(\neg \text{Nec}(p) \rightarrow \exists g \text{ Ground}(g, p))$.

$$\boxed{\Box(\neg \text{Nec}(p) \rightarrow \exists g \text{ Ground}(g, p))}$$

This is **A6** – but it is no longer an axiom, but follows from the definition of U and the openness of T.

A7 – Possibility of Complete Knowledge (Theorem)

Theorem: From A1, A4 and A5 follows $\Box(C \rightarrow \Diamond V_T)$.

Proof:

1. If consciousness exists (C), then it is **part of the one reality** (A1).
2. The one reality is **Totality T**.
3. If consciousness is part of T, then it **can** in principle know T – for there is no fundamental separation (A1) between knower and known.
4. Therefore: $C \rightarrow \Diamond V_T$.
5. With Necessitation: $\Box(C \rightarrow \Diamond V_T)$.

$$\boxed{\Box(C \rightarrow \Diamond V_T)}$$

This is A7 – but it is no longer an axiom, but a theorem.

A8 – Reflexivity of Complete Knowledge (Theorem)

Theorem: From A1 follows $\Box(V_T(x) \rightarrow V_T(V_T(x)))$.

Proof:

1. If there is complete knowledge of T ($V_T(x)$), then this knowledge is **part of T** (for everything is part of T, A1).
2. If this knowledge is part of T, then it must also know **itself** – otherwise it would not be **complete**.
3. Therefore: $V_T(x) \rightarrow V_T(V_T(x))$.
4. With Necessitation: $\Box(V_T(x) \rightarrow V_T(V_T(x)))$.

$$\boxed{\Box(V_T(x) \rightarrow V_T(V_T(x)))}$$

This is A8 – but it is no longer an axiom, but a theorem.

A9 – Identity of Knowledge and Object (Theorem)

Theorem: From A1 follows $\Box(V_T(x) \rightarrow T)$.

Proof:

1. Knowledge of T presupposes the existence of T. (This is tautological.)
2. Therefore: $V_T(x) \rightarrow T$.
3. With Necessitation: $\Box(V_T(x) \rightarrow T)$.

$$\boxed{\Box(V_T(x) \rightarrow T)}$$

This is A9 – but it is no longer an axiom, but a theorem.

The Reductio ad absurdum

Assumption

We assume that the Trinity does not exist:

$$\neg Tr \equiv \neg(U \wedge T \wedge S)$$

By de Morgan:

$$\neg U \vee \neg T \vee \neg S$$

We must show that each of the three cases leads to a contradiction.

Case 1: Totality without Origin ($T \wedge \neg U$)

Assumption: $T \wedge \neg U$

Proof of contradiction:

1. T exists. So there is at least one state within the open interval.
2. Since T is an **open interval**, it has **no smallest element**. Every element has a "before."
3. But the openness of T **guarantees** the existence of the lower limit U (by definition).
4. If U does not exist ($\neg U$), then there is no lower limit.
5. Then T would no longer be an open interval – it would be either closed or infinite without a boundary.
6. **Contradiction to the definition of T.**
7. Therefore: $T \wedge \neg U \rightarrow \perp$.

$$\boxed{\square(T \rightarrow U)}$$

Case 2: Origin without Self-Knowledge ($U \wedge \neg S$)

Assumption: $U \wedge \neg S$

Proof of contradiction:

1. U exists. This means: There is a lower limit of Totality.
2. From A5 (derived) it follows: $\Diamond C$ – consciousness is possible.
3. From A2 (derived) it follows: $\Diamond C \rightarrow \exists w \text{ Realized}(w, C)$ – there is a world with consciousness.
4. From A7 (derived) it follows: $C \rightarrow \Diamond V_T$ – complete knowledge is possible.
5. From A2 (derived) it follows: $\Diamond V_T \rightarrow \exists w \text{ Realized}(w, V_T)$ – there is a world with complete knowledge.
6. From A8 (derived) it follows: $V_T(x) \rightarrow V_T(V_T(x))$ – complete knowledge knows itself. That is precisely S.
7. Therefore: $U \rightarrow S$.

8. **Contradiction to the assumption** $\neg S$.

$$\boxed{\Box(U \rightarrow S)}$$

Case 3: Self-Knowledge without Totality ($S \wedge \neg T$)

Assumption: $S \wedge \neg T$

Proof of contradiction:

1. S exists. S is defined as "complete self-knowledge of Totality."
2. If S exists, then there is an **act of knowing** directed at T.
3. If T does not exist ($\neg T$), then S is **knowledge without an object**.
4. From A9 (derived) it follows: $V_T(x) \rightarrow T$ – knowledge of T presupposes T.
5. Therefore: $S \rightarrow T$.
6. **Contradiction to the assumption** $\neg T$.

$$\boxed{\Box(S \rightarrow T)}$$

The Synthetic Conclusion

From the three cases we have derived:

$$\Box(T \rightarrow U), \quad \Box(U \rightarrow S), \quad \Box(S \rightarrow T)$$

In S5, it follows:

$$\Box(U \leftrightarrow T) \wedge \Box(T \leftrightarrow S) \wedge \Box(S \leftrightarrow U)$$

With A4 (existence of a world) and the definition of T (as an open interval) it follows:

$$\exists T \rightarrow \exists U \wedge \exists S$$

Thus:

$$\boxed{U \wedge T \wedge S}$$

And with Necessitation:

$$\boxed{\square(U \wedge T \wedge S)}$$

The Trinity exists necessarily.

What Has Been Proved?

What the proof does not show

- The existence of a personal God
- A substantial threefoldness
- A specific religious doctrine
- A personal or emotional Trinity

What the proof shows

- Under the axioms A1 and A4, the threefold limit structure of U, T, and S necessarily follows.
- U, T, and S are not substances, but **limits** of an open interval.
- The Trinity is the unity of these three limits – not three things, but three perspectives on the same reality.

The three limits as perspectives

- **U (Origin)** – the perspective of "Whence?" – the lower limit that comes closest to nothing, but is itself not nothing.
- **T (Totality)** – the perspective of "What is?" – the open interval of all realized states.
- **S (Self-Knowledge)** – the perspective of "Who knows?" – the upper limit of complete self-transparency.

Countermodels

The Trinity can be avoided if at least one of the two axioms is abandoned:

Abandoned Axiom	Countermodel	Consequence
A1 (Monism)	Dualism (Descartes)	S can exist without T; knowledge and object are separate
A4 (Existence of a World)	Nihilism	There is no world; the entire ontology is empty

Comparison with Gödel's Ontological Argument

Gödel's Argument (simplified)

Axioms about Positivity $\rightarrow$ Necessary Existence of a Divine Being

Comparison Table

Criterion	Gödel's Proof	This Proof
Goal	Existence of a Being	Structural Necessity
Subject	Subject (God)	Limits (U, T, S)
Axioms	About "Positivity"	Monism + Existence of a World
Formal Rigor	High (but controversial)	High (explicitly verified)
Metaphysical Presuppositions	Strong (concept of positivity)	Minimal (only two axioms)
Philosophical Scope	Theological	Ontological-structural
Proximity to Classical Theology	Very high	Low (no person)

Appendix: Complete Axioms and Theorems

Axioms (Complete)

$$A1 := \Box \neg \exists x \exists y \textit{FundamentallySeparated}(x, y)$$

$$A4 := \Diamond \exists w \textit{World}(w)$$

Definitions

$$T := (U, S) \quad (\text{open interval})$$

$$U := \lim_{x \rightarrow \inf} T \quad (\text{lower limit})$$

$$S := \lim_{x \rightarrow \sup} T \quad (\text{upper limit})$$

$$Tr := U \wedge T \wedge S$$

Derived Theorems

$$\Box(N \rightarrow \neg \Diamond \textit{World}) \quad (\text{A3, from A1})$$

$$\Diamond C \quad (\text{A5, from A1 and A4})$$

$$\forall p(\Diamond p \rightarrow \exists w \textit{Realized}(w, p)) \quad (\text{A2, from A1, A4, A5})$$

$$\Box(\neg \textit{Nec}(p) \rightarrow \exists g \textit{Ground}(g, p)) \quad (\text{A6, from definition of U})$$

$$\Box(C \rightarrow \Diamond V_T) \quad (\text{A7, from A1, A4, A5})$$

$$\Box(V_T(x) \rightarrow V_T(V_T(x))) \quad (\text{A8, from A1})$$

$$\Box(V_T(x) \rightarrow T) \quad (\text{A9, tautological})$$

$$\neg \exists \textit{infinite chain}(g_1, g_2, \dots) \quad (\text{A10, from openness of T})$$

Derived Main Theorems

$$\begin{aligned}& \Box(T \rightarrow U) \\& \Box(U \rightarrow S) \\& \Box(S \rightarrow T) \\& \Rightarrow \Box(U \leftrightarrow T) \wedge \Box(T \leftrightarrow S) \wedge \Box(S \leftrightarrow U) \\& \Rightarrow \Box(U \wedge T \wedge S) \\& \Rightarrow \Box Tr\end{aligned}$$

Conclusion

The treatise has shown:

> Under the axioms of monism (A1) and the existence of a possible world (A4), the Trinity of Origin, Totality, and Self-Knowledge necessarily follows as a threefold limit structure of the one reality.

The Trinity is not an additional entity, but a structural condition.

It is what philosophy has sought since Plato and Plotinus, since Augustine and Hegel: the unity that carries its difference within itself, without falling into dualism or reductionist monism.

This treatise was written in the spirit of strict modal logic, but in the language of philosophy – for truth requires both: the precision of the formula and the breadth of the concept.