

The Minimality of the Superposition Axioms

A Metalogical Investigation into the Independence of ASP1–ASP5

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Zusammenfassung

In modal5.md it was shown: From the basic axioms A1, A4, A11, A12, A13, A14, A15, the Trinitarian formula does not follow in pure S5. The extension by the superposition axioms ASP1–ASP5 is sufficient. The question of whether it is also minimal remained open. The present treatise answers this question. It shows: ASP1 is derivable from ASP2 in S5 and is therefore redundant. The minimal subset for the derivation of the Trinitarian formula is {ASP3, ASP4, ASP5}. The full axiomatics is minimal for the superposition intuition, not for the target statement alone. The concept of minimality thus proves to be relative: it depends on what is to be derived and on the intuition to be formalized.

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Past and future are horizons of knowledge.

The present is the locus of reality.

Nothing that was, is lost.

Preface: What This Treatise Is – And What It Is Not

This treatise explicitly does not aim to prove the truth of a metaphysical or theological claim.

It investigates the **logical structure of an axiomatic system**. In `modal5.md` it was shown: From the basic axioms A1, A4, A11, A12, A13, A14, A15, the Trinitarian formula $\Box\forall x Tr(x)$ does **not** follow in pure S5. The extension by the superposition axioms ASP1–ASP5 is **sufficient**. The question remained open: Is it also **minimal**? Is each of the five axioms indispensable?

The present investigation answers this question. It shows: The superposition axioms are **not all indispensable**. ASP1 is derivable from ASP2. The minimal subset for the derivation of the Trinitarian formula is $\{ASP3, ASP4, ASP5\}$.

The investigation is therefore **metalogical**: It determines the **dependency structure** of a sufficient axiomatics. It does not show that this axiomatics is the only possible one – other extensions could also be sufficient. It only shows: **Within the extension chosen in `modal5.md`, the dependency structure is as it is.**

The central claim of this treatise is:

For the derivation of the Trinitarian formula $\Box\forall x Tr(x)$, the minimal subset of the superposition axioms is $\{ASP3, ASP4, ASP5\}$. ASP1 is derivable from ASP2. ASP2 is not necessary for the target statement, but is essential for the superposition intuition.

We stand at the beginning of infinity. Other axiomatizations are possible. What this treatise provides is a **map of dependencies** within the structure proposed in `modal5.md`.

Introduction: Three Forms of Minimality

The term “minimality” is ambiguous. Before we begin the actual investigation, we must distinguish precisely.

Redundancy

A set of axioms Σ is **redundant** if there is an axiom $\varphi \in \Sigma$ that is derivable from $\Sigma \setminus \{\varphi\}$:

$$\Sigma \setminus \{\varphi\} \vdash \varphi$$

Redundancy is a **defect**: The redundant axiom contributes nothing to the derivational strength and can be removed without weakening the theory.

Independence

A set of axioms Σ is **independent** if for **no** axiom $\varphi \in \Sigma$:

$$\Sigma \setminus \{\varphi\} \vdash \varphi$$

Independence is the **stronger** property: No axiom is derivable from the others.

Important: Independence is **not** the same as minimality in the sense of power. An independent set of axioms can be larger than necessary if another, smaller set of axioms generates the same theory. Independence is an **internal** property of a given set of axioms.

Semantic Indispensability (Sufficient Minimality)

A set of axioms Σ is **semantically indispensable** (or **sufficiently minimal**) for a target statement φ if:

1. $\Sigma \vdash \varphi$ (sufficiency).
2. For every proper subset $\Sigma' \subsetneq \Sigma$: $\Sigma' \not\vdash \varphi$ (indispensability).

This property is stronger than independence. It does not refer to the derivability of individual axioms from the others, but to the **derivability of the target statement** from subsets.

What This Treatise Shows

We show the **third** form: the **semantic indispensability** of the superposition axioms for the Trinitarian formula.

More precisely: For each relevant axiom we construct an S5 model that satisfies the remaining axioms, but in which $\neg \Box \forall x Tr(x)$ holds.

The existence of these models shows: The affected axioms are indispensable for the derivation of $\Box \forall x Tr(x)$ – at least within the axiomatics chosen in `modal5.md`.

The Methodological Self-Restriction

We do **not** show:

- that the superposition axioms are true,
- that there is no other sufficient extension,
- that there is no other minimal subset of ASP1–ASP5 (if the axioms are formulated differently).

We show **only**: In the version chosen in `modal5.md`, the dependency structure is as it is. This is an **independence result** in the sense of the metalogical tradition.

Formal Language and Logical Framework

Modal Logic S5

We work in first-order modal logic with identity in system S5:

- **Necessity:** $\Box p$
- **Possibility:** $\Diamond p := \neg \Box \neg p$

Axioms of S5:

- (K) $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$
- (T) $\Box p \rightarrow p$
- (4) $\Box p \rightarrow \Box \Box p$
- (5) $\Diamond p \rightarrow \Box \Diamond p$

Inference Rules:

- **Modus Ponens:** From p and $p \rightarrow q$, infer q .
- **Necessitation:** From p (derivable), infer $\Box p$.

Tableau Rules for S5:

$(\neg\Box)$	$\frac{\neg\Box A}{\Diamond\neg A}$
$(\Diamond)$	$\frac{\Diamond A}{A@w_{\text{new}}} \quad (\text{new world})$
$(\Box)$	$\frac{\Box A@w}{A@v} \quad \text{for every already existing world } v$
$(\neg\forall)$	$\frac{\neg\forall x A}{\exists x \neg A}$
$(\exists)$	$\frac{\exists x A}{A[a/x]} \quad (\text{a new})$
$(\forall)$	$\frac{\forall x A}{A[a/x]} \quad (\text{a arbitrary})$

Predicate Logic

We use classical first-order predicate logic with identity (=) and the usual quantifiers ($\forall, \exists$).

Models

An **S5 model** is a tuple $\mathcal{M} = (W, R, I)$ with:

- W : a non-empty set of worlds.
- $R \subseteq W \times W$: an **equivalence relation** (S5 frame).
- I : an interpretation function.

An **S5+SP model** is an S5 model that additionally satisfies the superposition axioms ASP1–ASP5.

The Basic Axioms (Repetition from modal5.md)

We repeat the basic axioms to make the treatise self-contained.

A1 – Monism

$$\boxed{A1 := \Box \neg \exists x \exists y \text{FundamentallySeparated}(x, y)}$$

A4 – Existence of a Possible Reality

$$\boxed{A4 := \Diamond \exists w \text{World}(w)}$$

A11 – Transcendental Bridge

$$A11 := \forall w (\text{WorldSeparatedFromConsciousness}(w) \leftrightarrow \neg \exists c (\text{Consciousness}(c) \wedge c(w)))$$

A12 – Experienceability and Realization

$$A12 := \forall p. \text{Experienceable}(p) \leftrightarrow \exists w. \text{Realized}(w, p)$$

A13, A14, A15 – The Three Implications of the Cases

$$A13 := \Box \forall x (T(x) \rightarrow U(x))$$

$$A14 := \Box \forall x (U(x) \rightarrow S(x))$$

$$A15 := \Box \forall x (S(x) \rightarrow T(x))$$

Definition of the Limit Structure

$$T := \{x \mid U < x < S\}$$

$$U := \lim_{x \rightarrow \inf} T$$

$$S := \lim_{x \rightarrow \sup} T$$

$$Tr := U \wedge T \wedge S$$

The Superposition Axioms (Repetition from modal5.md)

ASP1 – Consciousness in the Superposition

$$ASP1 := C@w_{\text{super}}$$

ASP2 – Uniqueness of C

$$ASP2 := \forall v (w_{\text{super}} Rv \rightarrow (C@v \leftrightarrow v = w_{\text{super}}))$$

ASP3 – The Superposition Contains All Tr-Properties

$$ASP3 := \forall x Tr(x)@w_{\text{super}}$$

ASP4 – Transfer to All Worlds

$$ASP4 := \forall v (w_{\text{super}} Rv \rightarrow \forall x Tr(x)@v)$$

ASP5 – Universal Reachability

$$ASP5 := \forall v (v \neq w_{\text{super}} \rightarrow w_{\text{super}} Rv)$$

The Proof Strategy

Goal

We want to show: For certain axioms of the superposition extension, there exists an S5 model $\mathcal{M}$ that satisfies the remaining axioms, but in which $\neg\Box\forall x Tr(x)$ holds.

Construction Principle

Each model violates **exactly one** of the superposition properties. All other superposition properties remain satisfied. The violation is sufficient to undermine the necessity of the Trinitarian formula.

Important Preliminary Remark: For $\neg\Box\forall x Tr(x)$ to hold in $\mathcal{M}$, it suffices to construct a world w^* in which $\neg\forall x Tr(x)$ holds – i.e., in which there is an individual a with $\neg T(a) \vee \neg U(a) \vee \neg S(a)$.

A First Result: ASP2 Implies ASP1 in S5

From ASP2, by instantiation $v := w_{\text{super}}$:

$$w_{\text{super}} R w_{\text{super}} \rightarrow (C@w_{\text{super}} \leftrightarrow w_{\text{super}} = w_{\text{super}})$$

In S5, reflexivity holds: $w_{\text{super}} R w_{\text{super}}$. And identity: $w_{\text{super}} = w_{\text{super}}$. Therefore:

$$C@w_{\text{super}}$$

That is ASP1. **ASP1 is therefore derivable from ASP2.** In the full axiomatics, ASP1 is **redundant**.

Consequence for the Strategy

The strategy “one model that violates exactly one axiom” is **not** feasible for all pairs. For ASP1 and ASP2: ASP2 implies ASP1 in S5. Therefore ASP1 is **redundant**.

Refined Goal: We show the **indispensability of ASP3, ASP4, ASP5** for the derivation of the Trinitarian formula. For ASP1, we show that in the version of `modal5.md` it is **redundant**. For ASP2, we show that it is not necessary for the target statement, but is essential for the superposition intuition.

Part A: Minimal Subset for the Target Statement

Theorem A1: $\{ASP3, ASP4, ASP5\}$ is Sufficient

Theorem: In S5:

$$\{ASP3, ASP4, ASP5\} \vdash \Box\forall x Tr(x)$$

Proof: Let $\mathcal{M} = (W, R, I)$ be an S5 model with ASP3, ASP4, ASP5. Let $w \in W$ be arbitrary.

- **Case 1:** $w = w_{\text{super}}$. By ASP3: $\mathcal{M}, w \models \forall x Tr(x)$.
- **Case 2:** $w \neq w_{\text{super}}$. By ASP5: $w_{\text{super}} R w$. By ASP4: $\mathcal{M}, w \models \forall x Tr(x)$.

In both cases, $\mathcal{M}, w \models \forall x Tr(x)$. Since w was arbitrary:

$$\mathcal{M} \models \Box \forall x Tr(x)$$

q.e.d.

Theorem A2: No Proper Subset of $\{ASP3, ASP4, ASP5\}$ is Sufficient

We show three countermodels.

Countermodel for $\{ASP4, ASP5\}$ (without ASP3)

Let $\mathcal{M}_3 = (W, R, I)$ with:

- $W = \{w_{\text{super}}, w^*\}$
- $R = W \times W$ (universal)
- I :
 - $Tr(x)@w_{\text{super}}$: false for an individual a .
 - $Tr(x)@w^*$: true for all x .

Verification:

- ASP4 satisfied: For all v with $w_{\text{super}}Rv$, $\forall x Tr(x)@v$ holds. In w^* , $\forall x Tr(x)$ holds, so ASP4 is satisfied.
- ASP5 satisfied: For all $v \neq w_{\text{super}}$, $w_{\text{super}}Rv$ holds.
- ASP3 violated: $\neg Tr(a)@w_{\text{super}}$.
- **Target statement:** In w_{super} , $\neg Tr(a)$ holds, hence $\neg \forall x Tr(x)$. Therefore $\neg \Box \forall x Tr(x)$.

Thus ASP3 is indispensable.

Countermodel for $\{ASP3, ASP5\}$ (without ASP4)

Let $\mathcal{M}_4 = (W, R, I)$ with:

- $W = \{w_{\text{super}}, w_1\}$
- $R = W \times W$ (universal)
- I :
 - $Tr(x)@w_{\text{super}}$: true for all x .
 - $Tr(x)@w_1$: false for a .

Verification:

- ASP3 satisfied: $\forall x Tr(x)@w_{\text{super}}$.
- ASP4 violated: $w_{\text{super}}Rw_1$, but $\neg Tr(a)@w_1$.
- ASP5 satisfied: For all $v \neq w_{\text{super}}$ (i.e., for $v = w_1$), $w_{\text{super}}Rw_1$ holds.
- **Target statement:** In w_1 , $\neg Tr(a)$ holds, hence $\neg \forall x Tr(x)$. Therefore $\neg \Box \forall x Tr(x)$.

Thus ASP4 is indispensable.

Countermodel for $\{ASP3, ASP4\}$ (without ASP5)

Let $\mathcal{M}_5 = (W, R, I)$ with:

- $W = \{w_{\text{super}}, w_1\}$
- $R = \{(w_{\text{super}}, w_{\text{super}}), (w_1, w_1)\}$ (two isolated worlds)
- I :
 - $Tr(x)@w_{\text{super}}$: true for all x .
 - $Tr(x)@w_1$: false for a .

Verification:

- ASP3 satisfied: $\forall x Tr(x)@w_{\text{super}}$.
- ASP4 satisfied: The only world v with $w_{\text{super}}Rv$ is w_{super} itself, and there $\forall x Tr(x)$ holds.
- ASP5 violated: $w_1 \neq w_{\text{super}}$, but $w_{\text{super}}Rw_1$ does not hold.
- **Target statement:** In w_1 , $\neg Tr(a)$ holds, hence $\neg \forall x Tr(x)$. Therefore $\neg \Box \forall x Tr(x)$.

Thus ASP5 is indispensable.

Theorem A3: $\{ASP3, ASP4, ASP5\}$ is Minimal

Theorem: The set $\{ASP3, ASP4, ASP5\}$ is a minimal subset of $\{ASP1, \dots, ASP5\}$ that derives $\Box \forall x Tr(x)$.

Proof: From Theorem A1, sufficiency follows. From Theorem A2, the indispensability of each of the three axioms follows: No proper subset of $\{ASP3, ASP4, ASP5\}$ derives $\Box \forall x Tr(x)$. Hence the set is minimal. q.e.d.

Part B: Role of ASP1 and ASP2

ASP1 and ASP2 Are Not Necessary for the Target Statement

Theorem B1: $\{ASP3, ASP4, ASP5\} \vdash \Box \forall x Tr(x)$, hence ASP1 and ASP2 are not required for the target statement.

Proof: Directly from Theorem A1. q.e.d.

ASP2 Implies ASP1 in S5

Theorem B2: In S5:

$$\{ASP2, \text{reflexivity}\} \vdash ASP1$$

Proof: From ASP2, by instantiation $v := w_{\text{super}}$:

$$w_{\text{super}}Rw_{\text{super}} \rightarrow (C@w_{\text{super}} \leftrightarrow w_{\text{super}} = w_{\text{super}})$$

In S5, reflexivity holds: $w_{\text{super}}Rw_{\text{super}}$. And identity: $w_{\text{super}} = w_{\text{super}}$. Therefore:

$$C@w_{\text{super}}$$

That is ASP1. q.e.d.

Consequence: ASP1 is Redundant

Theorem B3: In S5, ASP1 is derivable from ASP2. Hence ASP1 is redundant in the full axiomatics $\{ASP1, \dots, ASP5\}$.

Proof: Directly from Theorem B2. q.e.d.

Role of ASP1 and ASP2 for the Superposition Intuition

Theorem B4: ASP1 and ASP2 are not redundant for the **superposition intuition**, even though they are not required for the target statement $\Box\forall x Tr(x)$.

Justification: The superposition intuition states that consciousness (C) is the self-reflexive moment of the superposition. This intuition is formalized by ASP1 and ASP2:

- ASP1: C holds in the superposition.
- ASP2: C holds **only** in the superposition.

Without ASP1 and ASP2, the superposition intuition would not be fully formalized. The axioms ASP3–ASP5 alone say nothing about C .

Conclusion: There are two levels of minimality:

1. **Minimality for the target statement:** $\{ASP3, ASP4, ASP5\}$ is minimal.
2. **Minimality for the superposition intuition:** $\{ASP1, \dots, ASP5\}$ is minimal (apart from the redundancy of ASP1).

Part C: The Dependency Structure of the Superposition Axioms

The Structure in a Table

Axiom	Function	Necessary for target?	Derivable from others?
ASP1	C in w_{super}	No	Yes (from ASP2)
ASP2	C only in w_{super}	No	No
ASP3	Tr in w_{super}	Yes	No
ASP4	Tr in all reachable worlds	Yes	No
ASP5	Universal reachability	Yes	No

The Dependency Structure in a Diagram

ASP1 $\leftarrow$ ASP2 (ASP2 implies ASP1) / / / / [not necessary for target]

ASP3 $\rightarrow$ x Tr(x) (jointly sufficient) ASP4 ASP5

text

Interpretation

The dependency structure shows:

- **ASP2 is independent** and **stronger** than ASP1: It implies ASP1.
- **ASP1 is redundant** in the full axiomatics, but useful for the clarity of the intuition.

- **ASP3, ASP4, ASP5** form a **minimal sufficient subset** for the target statement.
- **ASP3, ASP4, ASP5** are **mutually independent**.

Part D: Consequences for modal5.md

The Axiomatics in modal5.md Is Not Minimal for the Target Statement

Consequence D1: The superposition axioms in modal5.md contain redundant elements (ASP1) and elements not required for the target statement (ASP1, ASP2).

Consequence D2: If the goal is only the derivation of $\Box\forall x Tr(x)$, then the minimal extension is $\{ASP3, ASP4, ASP5\}$.

Consequence D3: If the goal is the complete formalization of the superposition intuition, then the extension $\{ASP1, \dots, ASP5\}$ is appropriate, with the redundancy of ASP1.

The Metalogical Significance

Consequence D4: The distinction between “minimality for a goal” and “minimality for an intuition” is **central** to metalogical analysis. An axiomatics can be minimal for a goal and non-minimal for an intuition – and vice versa.

Consequence D5: The analysis shows that the **choice of axioms** depends not only on the target statement but also on the **intuition** to be formalized. The superposition intuition requires ASP1 and ASP2, even though these are not necessary for the target statement.

Open Questions

Question D6: Is there another formulation of the superposition intuition in which ASP1 and ASP2 are **indispensable**?

Question D7: Is the separation between ASP1/ASP2 (consciousness) and ASP3/ASP4/ASP5 (*Tr*) **systematic** or accidental?

Question D8: Could the superposition intuition be **formulated differently**, so that the target statement follows from a **smaller** axiomatics?

Part E: What Has Not Been Shown

No Statement About Other Extensions

This treatise has **not** shown that the superposition axioms are the **only** sufficient extension. Other extensions of S5 could also be sufficient, with different dependency structures.

Consequence E1: The non-uniqueness of the extension is the subject of **Modal 7**.

No Statement About the Truth of the Axioms

This treatise has **not** shown that the superposition axioms are **true**. It has only shown that they are **independent** (in the restricted sense).

Consequence E2: The question of the truth of the axioms is **not** a metalogical question. It is a matter for metaphysics, physics, or hermeneutics.

No Statement About Minimality in an Absolute Sense

This treatise has **not** shown that the superposition axioms are minimal in an absolute sense. It has only shown that they are minimal relative to a target statement or an intuition.

Consequence E3: The concept of “minimality” is **relative**. It depends on what is to be derived.

We Stand at the Beginning of Infinity

This treatise has clarified **one** aspect of the dependency structure of the superposition axioms. It has **not** answered the question of whether these axioms are the right choice for formalizing the Trinitarian structure. It has **not** answered the question of whether the Trinitarian structure exists at all. It has only shown: **Within the axiomatics chosen in `modal5.md`, the dependency structure is as it is.**

Conclusion: What Has Been Shown?

The present investigation has shown:

1. **ASP2 implies ASP1 in S5.** ASP1 is redundant in the full axiomatics.
2. **The minimal subset for the target statement $\Box\forall x Tr(x)$ is $\{ASP3, ASP4, ASP5\}$.** ASP1 and ASP2 are not necessary for the target statement.
3. **ASP3, ASP4, and ASP5 are mutually independent.** None is derivable from the others.
4. **The full axiomatics $\{ASP1, \dots, ASP5\}$ is minimal for the superposition intuition,** not for the target statement alone.
5. **The concept of minimality is relative.** It depends on what is to be derived and on the intuition to be formalized.

The central claim of this treatise is:

The superposition axioms in the version of `modal5.md` are not all indispensable for the derivation of the Trinitarian formula. The minimal subset for this derivation is $\{ASP3, ASP4, ASP5\}$. ASP1 is derivable from ASP2. The full axiomatics is minimal for the superposition intuition, not for the target statement.

The treatise is thus a **refinement** of `modal5.md`. It shows which axioms play which role, and it strictly distinguishes between:

- **Minimality for the target statement.**
- **Minimality for the superposition intuition.**

The question of whether the superposition axioms are the right choice for formalizing the Trinitarian structure remains open. It is the subject of **Modal 7** (Non-uniqueness).

Appendix A: The Formal Structure in an Overview

Axioms (Complete)

$$\begin{aligned}
A1 &:= \Box \neg \exists x \exists y \textit{FundamentallySeparated}(x, y) \\
A4 &:= \Diamond \exists w \textit{World}(w) \\
A11 &:= \forall w (\textit{WorldSeparatedFromConsciousness}(w) \leftrightarrow \neg \exists c (\textit{Consciousness}(c) \wedge c(w))) \\
A12 &:= \forall p. \textit{Experienceable}(p) \leftrightarrow \exists w. \textit{Realized}(w, p) \\
A13 &:= \Box \forall x (T(x) \rightarrow U(x)) \\
A14 &:= \Box \forall x (U(x) \rightarrow S(x)) \\
A15 &:= \Box \forall x (S(x) \rightarrow T(x)) \\
ASP1 &:= C@w_{\text{super}} \\
ASP2 &:= \forall v (w_{\text{super}} R v \rightarrow (C@v \leftrightarrow v = w_{\text{super}})) \\
ASP3 &:= \forall x \textit{Tr}(x)@w_{\text{super}} \\
ASP4 &:= \forall v (w_{\text{super}} R v \rightarrow \forall x \textit{Tr}(x)@v) \\
ASP5 &:= \forall v (v \neq w_{\text{super}} \rightarrow w_{\text{super}} R v)
\end{aligned}$$

Definitions

$$\begin{aligned}
\textit{WorldSeparatedFromConsciousness}(w) &:= \neg \exists c (\textit{Consciousness}(c) \wedge c(w)) \\
\textit{Experienceable}(p) &:= \exists w. (\textit{Consciousness}(w) \wedge \textit{Realized}(w, p)) \\
T &:= \{x \mid U < x < S\} \\
U &:= \lim_{x \rightarrow \inf} T \\
S &:= \lim_{x \rightarrow \sup} T \\
\textit{Tr} &:= U \wedge T \wedge S
\end{aligned}$$

Derived Theorems

$$\begin{aligned}
\{ASP2\} &\vdash ASP1 \quad (\text{in S5}) \\
\{ASP3, ASP4, ASP5\} &\vdash \Box \forall x \textit{Tr}(x) \\
\{ASP1, \dots, ASP5\} &\vdash \Box \forall x \textit{Tr}(x) \\
\{ASP3, ASP4, ASP5\} &\text{ is minimal for } \Box \forall x \textit{Tr}(x) \\
\{ASP1, \dots, ASP5\} &\text{ is minimal for the superposition intuition}
\end{aligned}$$

Appendix B: The Countermodels in an Overview

Model	W	R	ASP1	ASP2	ASP3	ASP4	ASP5	Target state- ment
$\mathcal{M}_3$	$\{w_{\text{super}}, w^*\}$		universal					
$\mathcal{M}_4$	$\{w_{\text{super}}, w_1\}$		universal					

$\mathcal{M}_5$ $\{w_{\text{super}}, w_1\}$ two
isolated
classes

Explanation: The table shows three models that each satisfy the full axiomatics except for one axiom, and in which the target statement $\Box\forall x Tr(x)$ does **not** hold. Thus each of the three axioms ASP3, ASP4, ASP5 is indispensable.

Appendix C: The Role of ASP1 and ASP2 in an Overview

Axiom	Function	Necessary for target?	Redundant?
ASP1	C in w_{super}	No	Yes (from ASP2)
ASP2	C only in w_{super}	No	No

Explanation: ASP1 and ASP2 formalize the role of consciousness in the superposition. They are not necessary for the target statement $\Box\forall x Tr(x)$, but they are essential for the superposition intuition.

Afterword

You have read this treatise. Perhaps the formal language remained foreign to you; perhaps you found it clarifying. Both are perfectly acceptable. For this text does not seek to proclaim a final truth, but rather to render a movement of thought comprehensible.

The investigation has shown: The question of minimality is **relative**. It depends on what is to be derived. Relative to the Trinitarian formula, the minimal axiomatics is $\{ASP3, ASP4, ASP5\}$. Relative to the superposition intuition, the minimal axiomatics is $\{ASP1, \dots, ASP5\}$, with the redundancy of ASP1.

This relativity is **not a defect**. It is an **insight**: The concept of minimality is not absolute. It depends on goal and intuition.

What remains is the insight that the Trinitarian structure **does not** follow from monism alone – it requires additional structure. The superposition axioms are **one** such structure. Whether they are the right one remains open.

One final question:

Do we dare the leap into openness to the limit – even though the limit itself is not formally reachable?

This treatise was written in the spirit of rigorous modal logic, yet in the language of philosophy – for truth requires both: the precision of the formula and the breadth of the concept.