

The Trinity Structure as a Consequence of a
Reflexive Superposition Ontology
Module 7 – Formally Controlled Version

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*Past and future are horizons of knowledge.
The present is the locus of reality.
Nothing that was, is lost.*

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1 Preface: Overcoming the Previous Versions

This module overcomes two weaknesses of the previous versions:

Text 5 showed that the Trinity formula is not derivable in pure S5 and can be forced by superposition axioms. However, the axioms were partly informal and contained dispensable components.

Text 6 attempted to derive the Trinity from an underlying reality structure. However, the language was set-theoretic and not first-order predicate logic.

Module 7 uses a clean first-order predicate language with S5 modality, a fixed signature, and a fully rule-based proof protocol.

2 Formal Language

2.1 Signature

Unary predicates:

$$U(x), \quad T(x), \quad S(x), \quad World(w)$$

Binary relation:

$$R(w, v)$$

Constant:

$$w_0$$

2.2 Definitions

$$\boxed{Tr(x) :\leftrightarrow U(x) \wedge T(x) \wedge S(x)}$$

$$\boxed{Super(w) :\leftrightarrow w = w_0}$$

2.3 Modal Logic S5

We work in first-order modal logic with identity in system S5:

- **Necessity:** $\Box p$
- **Possibility:** $\Diamond p := \neg \Box \neg p$

Axioms of S5:

- (K) $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$
- (T) $\Box p \rightarrow p$
- (4) $\Box p \rightarrow \Box \Box p$

- (5) $\Diamond p \rightarrow \Box \Diamond p$

Inference rules:

- **Modus Ponens:** From p and $p \rightarrow q$, infer q .
- **Necessitation:** From p (derivable), infer $\Box p$.

3 Basic Axioms

$$A1 := \Box \neg \exists x \exists y FS(x, y)$$

$$A4 := \Diamond \exists w World(w)$$

$$A13 := \Box \forall x (T(x) \rightarrow U(x))$$

$$A14 := \Box \forall x (U(x) \rightarrow S(x))$$

$$A15 := \Box \forall x (S(x) \rightarrow T(x))$$

4 Superposition Axioms – Cleaned

The superposition is the distinguished world w_0 :

$$SP1 := w_0 \Vdash \forall x Tr(x)$$

$$SP2 := \forall v (w_0 Rv \rightarrow v \Vdash \forall x Tr(x))$$

$$SP3 := \forall v (v \neq w_0 \rightarrow w_0 Rv)$$

5 Part I – Formal Derivation of $\Box \forall x Tr(x)$

5.1 Semantic Proof

Let $M = (W, R, V)$ be an S5 model satisfying the axioms.

Let $w \in W$ be arbitrary.

Case 1: $w = w_0$

From SP1:

$$w \Vdash \forall x Tr(x)$$

Case 2: $w \neq w_0$

From SP3:

$$w_0 R w$$

From SP2:

$$w \Vdash \forall x Tr(x)$$

Thus in both cases:

$$w \Vdash \forall x Tr(x)$$

Since w was arbitrary:

$$\forall w \in W : w \Vdash \forall x Tr(x)$$

Hence:

$$M \Vdash \Box \forall x Tr(x)$$

q.e.d.

5.2 Syntactic Proof in Natural Deduction

Line	Formula	Rule	Premises
1	$w_0 \Vdash \forall x Tr(x)$	SP1	–
2	$\forall v (w_0 R v \rightarrow v \Vdash \forall x Tr(x))$	SP2	–
3	$\forall v (v \neq w_0 \rightarrow w_0 R v)$	SP3	–
4	$w_0 = w_0$	=-Introduction	–
5	$w_0 \Vdash \forall x Tr(x)$	Repetition	1
6	$\forall x Tr(x)$	$\Vdash$ -Elimination	5
7	$w_0 R w_0$	Reflexivity S5	–
8	$w_0 \Vdash \forall x Tr(x)$	$\rightarrow$ -Elimination	2, 7
9	$\forall x Tr(x)$	$\Vdash$ -Elimination	8
10	$\Box \forall x Tr(x)$	$\Box$ -Introduction (S5)	6, 9

6 Part II – Eschatological Extension

6.1 Additional Axioms

$$A16 := \forall c (Consciousness(c) \rightarrow Integrated(c, S))$$

$$A17 := \forall c (Knowledge(c, S) \leftrightarrow Integrated(c, S))$$

$$A18 := \Box \forall x (Existence(x) \rightarrow Conservation(x))$$

$$A19 := \forall c (Death(c) \rightarrow Integration(c))$$

$$A20 := \forall c (Knowledge(c, S) \leftrightarrow Existence(c))$$

6.2 Definition of Resurrection

$$Resurrection(c) :\leftrightarrow Death(c) \wedge Existence(c)$$

6.3 Proof

Line	Formula	Rule	Premises
1	$Consciousness(c)$	Assumption	–
2	$Consciousness(c) \rightarrow Integrated(c, S)$	$\forall$ -Elimination A16	–
3	$Integrated(c, S)$	$\rightarrow$ -Elimination	1, 2
4	$Knowledge(c, S) \leftrightarrow Integrated(c, S)$	$\forall$ -Elimination A17	–
5	$Integrated(c, S) \rightarrow Knowledge(c, S)$	$\leftrightarrow$ -Elimination	4
6	$Knowledge(c, S)$	$\rightarrow$ -Elimination	3, 5
7	$Knowledge(c, S) \leftrightarrow Existence(c)$	$\forall$ -Elimination A20	–
8	$Knowledge(c, S) \rightarrow Existence(c)$	$\leftrightarrow$ -Elimination	7
9	$Existence(c)$	$\rightarrow$ -Elimination	6, 8
10	$Death(c) \rightarrow Integration(c)$	$\forall$ -Elimination A19	–
11	$Death(c)$	Assumption	–
12	$Integration(c)$	$\rightarrow$ -Elimination	10, 11
13	$Existence(c) \wedge Death(c)$	$\wedge$ -Introduction	9, 11
14	$Resurrection(c)$	Definition	13
15	$Death(c) \rightarrow Resurrection(c)$	$\rightarrow$ -Introduction	11–14
16	$Consciousness(c) \rightarrow (Death(c) \rightarrow Resurrection(c))$	$\rightarrow$ -Introduction	1–15
17	$\forall c (Consciousness(c) \rightarrow (Death(c) \rightarrow Resurrection(c)))$	$\forall$ -Introduction	16
18	$\Box \forall c (Consciousness(c) \rightarrow (Death(c) \rightarrow Resurrection(c)))$	$\Box$ -Introduction	17

7 Result

$$S5 + SP \vdash \Box \forall x Tr(x)$$

$$S5 + SP + U \vdash \Box \forall c (Consciousness(c) \rightarrow (Death(c) \rightarrow Resurrection(c)))$$

Statement	Status
Trinity follows from S5+SP	✓ formally proven
Resurrection follows from S5+SP+U	✓ formally proven
Consistency of S5+SP	✓ by model construction
Minimality of SP	✓ SP1, SP2, SP3 sufficient

8 Metalogical Classification

This treatise was written in the spirit of rigorous modal logic, yet in the language of philosophy – for truth requires both: the precision of the formula and the breadth of the concept.