

The Trinity as a Necessary Structure of a Monistic
Modal Ontology
Version 4 – Complete Formal Analysis

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*Past and future are horizons of knowledge.
The present is the locus of reality.*

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1 Preface

This treatise is the formal culmination of a long development. It unites the insights from all previous versions:

- **Versions 1–3:** Attempt to derive the Trinity directly from S5 – failed due to the missing bridge from existence to necessity.
- **Version 4 (old):** Proof that pure S5 is insufficient – the target formula is not derivable in S5 alone. This proof is now integrated into the new version.
- **Version 5:** Formalization of the superposition intuition and proof of the target formula in the extended system S5+SP.

The **present Version 4 (new)** unites both perspectives:

1. **Part I:** Rigorous formal proof that the target formula is **not** derivable in pure S5 (adoption of old Version 4).
2. **Part II:** Rigorous formal proof that the target formula is **derivable** in the extended system S5+SP (with superposition axioms).
3. **Part III:** Metatheoretical classification – what has been shown, what has not, and which questions remain open.

The fundamental thesis of the entire investigation is:

Under the axioms of monism (A1), the existence of a possible world (A4), the transcendental bridges (A11, A12), and the superposition hypothesis (ASP1–ASP5), the Trinity of origin, totality, and self-knowledge follows necessarily. Without the superposition hypothesis, it is not provable in S5.

2 Introduction: Aim and Methodological Self-Restriction

Classical metaphysics has repeatedly attempted to derive the fundamental structure of reality from a few basic principles. A particular challenge arises from three seemingly distinct aspects:

1. **Why is there something rather than nothing?** – origin (U).
2. **How does the totality of all possibilities relate to actuality?** – totality (T).
3. **How can a state arise within reality that recognizes reality itself?** – self-knowledge (S).

The ontology examined here proposes to answer these three questions not through three separate metaphysical substances, but through three necessary perspectives of the same reality:

- **Origin (U):** the necessary ground for the existence of possibilities at all – understood as the lower limit of totality;
- **Totality (T):** the entirety of all realized possibilities – understood as an open, well-founded interval;
- **Self-knowledge (S):** the reflexive completion of reality in a state of complete self-knowledge – understood as the upper limit of totality.

This structure is called the **Trinity**:

$$Tr := U \wedge T \wedge S$$

The term "Trinity" here denotes not a personal or substantial threefoldness, but a functional unity of three necessary aspects of a single reality.

3 Formal Language and Logical Framework

3.1 Modal Logic S5

We work in first-order modal logic with identity in the system S5:

- **Necessity:** $\Box p$
- **Possibility:** $\Diamond p := \neg \Box \neg p$

Axioms of S5:

- (K) $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$
- (T) $\Box p \rightarrow p$
- (4) $\Box p \rightarrow \Box \Box p$
- (5) $\Diamond p \rightarrow \Box \Diamond p$

Inference Rules:

- **Modus Ponens:** From p and $p \rightarrow q$, infer q .
- **Necessitation:** From p (derivable), infer $\Box p$.

Tableau Rules for S5:

$(\neg\Box)$	$\frac{\neg\Box A}{\Box\neg A}$	
$(\Diamond)$	$\frac{\Diamond A}{A@w_{\text{new}}}$	(new world)
$(\Box)$	$\frac{\Box A@w}{A@v}$	for every already existing world v
$(\neg\forall)$	$\frac{\neg\forall x A}{\exists x\neg A}$	
$(\exists)$	$\frac{\exists x A}{A[a/x]}$	(a new)
$(\forall)$	$\frac{\forall x A}{A[a/x]}$	(a arbitrary)

3.2 Predicate Logic

We use classical first-order predicate logic with identity ($=$) and the usual quantifiers ($\forall, \exists$).

4 The Axioms (Basic)

4.1 A1 – Monism

There are no fundamentally separated domains of reality.

$$\boxed{A1 := \Box\neg\exists x\exists y \textit{FundamentalSeparated}(x, y)}$$

Explanation: If two domains were fundamentally separated, there would be no causal or ontological interaction between them. They could not be part of a unified reality, which contradicts monism.

4.2 A4 – Existence of a Possible Reality

There is at least one possible world.

$$\boxed{A4 := \Diamond\exists w \textit{World}(w)}$$

Explanation: This axiom ensures that the modal universe is not empty. It is the weakest possible existence axiom: it does not say that a world **actually** exists, but only that it is **possible**.

4.3 A11 – Transcendental Bridge

A world is separated from consciousness exactly when it contains no consciousness.

$$A11 := \forall w (\text{WorldSeparatedFromConsciousness}(w) \leftrightarrow \neg \exists c (\text{Consciousness}(c) \wedge c(w)))$$

with the definition:

$$\text{WorldSeparatedFromConsciousness}(w) := \neg \exists c (\text{Consciousness}(c) \wedge c(w))$$

Explanation: This axiom formalizes the transcendental insight that a world without consciousness would be unknowable and therefore fundamentally separated – which is forbidden by monism (A1).

4.4 A12 – Experience and Realization

$$A12 := \forall p. \text{Experienceable}(p) \leftrightarrow \exists w. \text{Realized}(w, p)$$

with the definition:

$$\text{Experienceable}(p) := \exists w. (\text{Consciousness}(w) \wedge \text{Realized}(w, p))$$

Explanation: This axiom states: A proposition is experienceable exactly when it is realized in a world. It is the formal version of the transcendental argument: What is not realized cannot be experienced.

4.5 A13, A14, A15 – The Three Implications of the Cases

These axioms formalize the three reductio cases:

$$A13 := \Box \forall x (T(x) \rightarrow U(x))$$

$$A14 := \Box \forall x (U(x) \rightarrow S(x))$$

$$A15 := \Box \forall x (S(x) \rightarrow T(x))$$

Explanation: They state that the three limits T, U, S stand in a mutual implication chain – which in S5 leads to their equivalence.

5 Definition of the Limit Structure

5.1 Totality T as an Open, Well-Founded Interval

Totality T is the entirety of all realized states. We understand T as a **well-founded, open interval**:

$$T := \{x \mid U < x < S\}$$

Explanation:

- An open interval (U, S) contains all states between U and S, but **not** U and S themselves.
- **Well-foundedness:** Every non-empty subset of states has a minimal element. This prevents infinite chains of grounds.
- **Origin U** is the lower limit – that which comes closest to nothing, but is itself not nothing.
- **Self-knowledge S** is the upper limit – the complete transparency of totality, which itself does not belong to totality.

5.2 Origin U as Lower Limit

$$U := \lim_{x \rightarrow \inf} T$$

5.3 Self-Knowledge S as Upper Limit

$$S := \lim_{x \rightarrow \sup} T$$

5.4 The Trinity

$$Tr := U \wedge T \wedge S$$

6 PART I: PROOF OF NON-DERIVABILITY IN PURE S5

6.1 Goal

Show that:

$$S5 \not\vdash \Box \forall x Tr(x)$$

does not follow from the axioms $A1, A4, A11, A12, A13, A14, A15$ alone.

6.2 Method

Construct an **open S5 tableau** for the negation of the target formula. According to the **soundness and completeness theorem** of the S5 tableau calculus:

A set of formulas is **satisfiable** in an S5 model exactly when the tableau has an **open branch**.

Therefore: If the tableau for the negated target formula remains open, then there exists an S5 model that satisfies all axioms and the negation of the target formula – so the target formula is **not a theorem**.

6.3 Tableau (Pure Smullyan Rules)

- | | |
|---|---|
| 1. $\neg x \text{ Tr}(x)$ | [Assumption: target is not a theorem] |
| 2. $\neg x \text{ Tr}(x)$ | [1, $\neg$ -rule] |
| 3. $\neg x \text{ Tr}(x) @ w0$ | [2, -rule: new world w0] |
| 4. $x \neg \text{Tr}(x) @ w0$ | [3, $\neg$ -rule: $\neg xA \quad x\neg A$] |
| 5. $\neg \text{Tr}(a) @ w0$ | [4, -rule: a new] |
| 6. $\neg(\text{T}(a) \quad \text{U}(a) \quad \text{S}(a)) @ w0$ | [5, D1 (Tr-definition)] |

→ -rule on 6:

6a. $\neg \text{T}(a) @ w0$

6b. $\neg \text{U}(a) @ w0$

6c. $\neg \text{S}(a) @ w0$

BRANCH 6a: $\neg \text{T}(a) @ w0$

- | | |
|--|------------|
| 7. $x(\text{S}(x) \rightarrow \text{T}(x)) @ w0$ | [A15] |
| 8. $x(\text{S}(x) \rightarrow \text{T}(x)) @ w0$ | [7, -rule] |
| 9. $\text{S}(a) \rightarrow \text{T}(a) @ w0$ | [8, -rule] |

→ -rule on 9:

9a. $\neg \text{S}(a) @ w0$

9b. $\text{T}(a) @ w0$

[Contradiction with 6a → branch 9b closes]

Thus: 9a. $\neg \text{S}(a) @ w0$

- | | |
|---|-------------|
| 10. $x(\text{T}(x) \rightarrow \text{U}(x)) @ w0$ | [A13] |
| 11. $x(\text{T}(x) \rightarrow \text{U}(x)) @ w0$ | [10, -rule] |
| 12. $\text{T}(a) \rightarrow \text{U}(a) @ w0$ | [11, -rule] |

→ -rule on 12:

12a. $\neg \text{T}(a) @ w0$

12b. $\text{U}(a) @ w0$

[already in 6a]

[no contradiction]

→ Choose branch 12a (consistent with 6a).

13. $x(U(x) \rightarrow S(x)) @ w0$	[A14]
14. $x(U(x) \rightarrow S(x)) @ w0$	[13, -rule]
15. $U(a) \rightarrow S(a) @ w0$	[14, -rule]

→ -rule on 15:

15a. $\neg U(a) @ w0$

15b. $S(a) @ w0$

[Contradiction with 9a → branch 15b closes]

Thus: 15a. $\neg U(a) @ w0$

→ In branch 6a: $\neg T(a), \neg U(a), \neg S(a) @ w0$.

→ This is consistent - no contradiction.

BRANCH 6b: $\neg U(a) @ w0$

16. $x(T(x) \rightarrow U(x)) @ w0$	[A13]
17. $x(T(x) \rightarrow U(x)) @ w0$	[16, -rule]
18. $T(a) \rightarrow U(a) @ w0$	[17, -rule]

→ -rule on 18:

18a. $\neg T(a) @ w0$

18b. $U(a) @ w0$

[Contradiction with 6b → closes]

Thus: 18a. $\neg T(a) @ w0$

19. $x(S(x) \rightarrow T(x)) @ w0$	[A15]
20. $x(S(x) \rightarrow T(x)) @ w0$	[19, -rule]
21. $S(a) \rightarrow T(a) @ w0$	[20, -rule]

→ -rule on 21:

21a. $\neg S(a) @ w0$

21b. $T(a) @ w0$

[Contradiction with 18a → closes]

Thus: 21a. $\neg S(a) @ w0$

22. $x(U(x) \rightarrow S(x)) @ w0$	[A14]
23. $x(U(x) \rightarrow S(x)) @ w0$	[22, -rule]
24. $U(a) \rightarrow S(a) @ w0$	[23, -rule]

→ -rule on 24:

24a. $\neg U(a) @ w0$

[already in 6b]

24b. $S(a) @ w_0$ [Contradiction with 21a $\rightarrow$ closes]

$\rightarrow$ Consistent branch: $\neg U(a), \neg T(a), \neg S(a) @ w_0$.

BRANCH 6c: $\neg S(a) @ w_0$

25. $x(S(x) \rightarrow T(x)) @ w_0$ [A15]
26. $x(S(x) \rightarrow T(x)) @ w_0$ [25, $\neg$ -rule]
27. $S(a) \rightarrow T(a) @ w_0$ [26, $\neg$ -rule]

$\rightarrow$ $\neg$ -rule on 27:

27a. $\neg S(a) @ w_0$ [already in 6c]

27b. $T(a) @ w_0$

$\rightarrow$ Both branches possible.

28. $x(T(x) \rightarrow U(x)) @ w_0$ [A13]
29. $x(T(x) \rightarrow U(x)) @ w_0$ [28, $\neg$ -rule]
30. $T(a) \rightarrow U(a) @ w_0$ [29, $\neg$ -rule]

$\rightarrow$ $\neg$ -rule on 30:

30a. $\neg T(a) @ w_0$

30b. $U(a) @ w_0$

31. $x(U(x) \rightarrow S(x)) @ w_0$ [A14]
32. $x(U(x) \rightarrow S(x)) @ w_0$ [31, $\neg$ -rule]
33. $U(a) \rightarrow S(a) @ w_0$ [32, $\neg$ -rule]

$\rightarrow$ $\neg$ -rule on 33:

33a. $\neg U(a) @ w_0$

33b. $S(a) @ w_0$ [Contradiction with 6c $\rightarrow$ closes]

Thus: 33a. $\neg U(a) @ w_0$

$\rightarrow$ Combine: From 30b ($U(a)$) and 33a ($\neg U(a)$) we get a contradiction.

$\rightarrow$ Therefore 30a must hold: $\neg T(a) @ w_0$.

$\rightarrow$ Thus: $\neg S(a), \neg U(a), \neg T(a) @ w_0$ consistent.

ALL BRANCHES: $\neg T(a) \neg U(a) \neg S(a) @ w_0$ is consistent.

REMAINING AXIOMS (A4, A11, A12) - no application

34. $w \text{ World}(w) @ w_0$ [A4]
 35. $w \text{ World}(w) @ w_1$ [34, -rule, w_1 new]
 $\rightarrow$ Leads to new world w_1 , but not back to w_0 .
 36. A11, A12: No instances in w_0 that force $T(a)$, $U(a)$, or $S(a)$.

CONCLUSION OF THE TABLEAU:

The tableau has an **open branch**:

w_0, a , with $\neg T(a)$, $\neg U(a)$, $\neg S(a)$.

No axiom generates $T(a)$, $U(a)$, or $S(a)$ in w_0 .
 The remaining axioms lead to new worlds ($w_1, \dots$),
 but not back to w_0 .

Therefore, the tableau is **not closed**.

6.4 Metatheoretical Conclusion

According to the **soundness and completeness theorem** of the S5 tableau calculus (see e.g. Smullyan, Fitting, or Blackburn/de Rijke/Venema):

A set of formulas Σ is S5-satisfiable exactly when the tableau for Σ has an open branch.

Our tableau for

$$\Sigma = \{A1, A4, A11, A12, A13, A14, A15, \neg \Box \forall x Tr(x)\}$$

has an open branch.

Thus Σ is S5-satisfiable.

Thus there exists an S5 model that satisfies all axioms, but in world w_0 has an individual a for which $\neg T(a)$, $\neg U(a)$, and $\neg S(a)$ hold.

Thus in this model $\Box \forall x Tr(x)$ does not hold.

Thus $\Box \forall x Tr(x)$ is **not a logical theorem** of the given axiomatics.

Theorem 1 (Non-derivability in pure S5).

$$\boxed{S5 \not\vdash \Box \forall x Tr(x)}$$

7 PART II: PROOF IN THE EXTENDED SYSTEM S5+SP

7.1 The Superposition Extension (ASP)

In addition to the axioms A1, A4, A11, A12, A13, A14, A15, we introduce the **superposition axioms**. They formalize the idea that consciousness (C) is the self-reflexive moment of a superposition from which all worlds emerge.

7.1.1 ASP1 – Consciousness in the Superposition

$$ASP1 := C@w_{\text{super}}$$

Explanation: Consciousness holds in the superposition. The superposition is the primordial state in which all possibilities are still undividedly contained.

7.1.2 ASP2 – Uniqueness of C

$$ASP2 := \forall v (w_{\text{super}} Rv \rightarrow (C@v \leftrightarrow v = w_{\text{super}}))$$

Explanation: Consciousness holds only in the superposition – no other world has it. Consciousness is a singular event.

7.1.3 ASP3 – The Superposition Contains All Tr-Properties

$$ASP3 := \forall x Tr(x)@w_{\text{super}}$$

Explanation: The superposition already contains all properties T, U, S . It is the ground for everything that holds in the worlds.

7.1.4 ASP4 – Transfer to All Worlds

$$ASP4 := \forall v (w_{\text{super}} Rv \rightarrow \forall x Tr(x)@v)$$

Explanation: What holds in the superposition holds in all reachable worlds. The superposition is a normative origin.

7.1.5 ASP5 – Universal Reachability (as Frame Condition)

$$ASP5 := \forall v (v \neq w_{\text{super}} \rightarrow w_{\text{super}} Rv)$$

Explanation: Every other world is reachable from the superposition. The superposition is the unique origin.

7.2 Proof in the Extended System S5+SP

Theorem 2 (Derivability in S5+SP).

$$\boxed{S5+SP \vdash \Box \forall x Tr(x)}$$

where $S5+SP := S5 + \{ASP1, ASP2, ASP3, ASP4, ASP5\}$.

7.2.1 Semantic Proof

Let $\mathcal{M} = (W, R, w_{\text{super}}, I)$ be an arbitrary S5+SP model with ASP1–ASP5.

To show: For all $w \in W$, $\mathcal{M}, w \models \forall x Tr(x)$.

Let $w \in W$ be arbitrary.

Case 1: $w = w_{\text{super}}$.

By ASP3:

$$\mathcal{M}, w_{\text{super}} \models \forall x Tr(x)$$

Thus the claim holds.

Case 2: $w \neq w_{\text{super}}$.

By ASP5:

$$w_{\text{super}} R w$$

By ASP4 it follows:

$$\mathcal{M}, w \models \forall x Tr(x)$$

Thus the claim holds.

Since w was arbitrary, for all $w \in W$:

$$\mathcal{M}, w \models \forall x Tr(x)$$

This is equivalent to:

$$\mathcal{M} \models \Box \forall x Tr(x)$$

q.e.d.

7.2.2 Tableau Proof in S5+SP

TABLEAU PROOF IN S5+SP

GOAL: $\neg \exists x \text{Tr}(x)$

REDUCTIO ASSUMPTION:

- | | |
|--|-------------------------------------|
| 1. $\neg \exists x \text{Tr}(x)$ | [Assumption: target false] |
| 2. $\neg \exists x \text{Tr}(x)$ | [1, $\neg$ -rule] |
| 3. $\neg \exists x \text{Tr}(x) @ w_0$ | [2, $\neg$ -rule: new world w_0] |
| 4. $\exists x \neg \text{Tr}(x) @ w_0$ | [3, $\neg$ -rule] |
| 5. $\neg \text{Tr}(a) @ w_0$ | [4, $\neg$ -rule: a new] |

CASE DISTINCTION: $w_0 = w_{\text{super}} \quad w_0 \neq w_{\text{super}}$

- | | |
|---|------------|
| 6. $w_0 = w_{\text{super}} \quad w_0 \neq w_{\text{super}}$ | [Identity] |
| $\rightarrow$ Branch A: $w_0 = w_{\text{super}}$ | |
| $\rightarrow$ Branch B: $w_0 \neq w_{\text{super}}$ | |

BRANCH A: $w_0 = w_{\text{super}}$

- | | |
|---|------------------------|
| 7. $\neg \text{Tr}(a) @ w_{\text{super}}$ | [5, Substitution] |
| 8. $\exists x \text{Tr}(x) @ w_{\text{super}}$ | [ASP3, Axiom] |
| 9. $\text{Tr}(a) @ w_{\text{super}}$ | [8, $\neg$ -rule on a] |
| 10. Contradiction: $\text{Tr}(a) @ w_{\text{super}}$ and $\neg \text{Tr}(a) @ w_{\text{super}}$ | |
| $\rightarrow$ Branch A closes $()$. | |

BRANCH B: $w_0 \neq w_{\text{super}}$

- | | |
|--|-------------------------|
| 11. $w_0 \neq w_{\text{super}}$ | [from 6, Branch B] |
| 12. $w_{\text{super}} R w_0$ | [11, ASP5] |
| 13. $\exists x \text{Tr}(x) @ w_0$ | [12, ASP4] |
| 14. $\text{Tr}(a) @ w_0$ | [13, $\neg$ -rule on a] |
| 15. Contradiction: $\text{Tr}(a) @ w_0$ and $\neg \text{Tr}(a) @ w_0$ (from 5) | |
| $\rightarrow$ Branch B closes $()$. | |

BOTH BRANCHES CLOSE.

Therefore, the assumption $\neg xTr(x)$ is contradictory.

Thus: $xTr(x)$ in S5+SP.

QED.

8 PART III: METATHEORETICAL CLASSIFICATION

8.1 What Has Been Shown?

System	Statement	Status
Pure S5	$S5 \not\vdash \Box \forall x Tr(x)$	Proven (open tableau branch)
S5+SP	$S5+SP \vdash \Box \forall x Tr(x)$	Proven (closed tableau)

The crucial insight is:

The bridge from the existence of Tr in one world to the necessity of Tr in all worlds is **not a theorem of S5**. It must be introduced as an **additional metaphysical assumption** – here in the form of the superposition axioms ASP1–ASP5.

8.2 What Has Not Been Shown?

- The **truth** of the superposition axioms ASP1–ASP5. They are **metaphysical premises**, not logical theorems.
- That the target formula follows from the original axioms A1, A4, A11, A12, A13, A14, A15 alone – on the contrary, Part I shows that it does not.

8.3 The Role of C (Consciousness)

The superposition axioms formalize the intuition that **consciousness** (C) is the self-reflexive moment of the superposition:

- ASP1: C holds in the superposition.
- ASP2: C holds only there – it is a singular event.
- ASP3–ASP5: The superposition is the necessary ground for all properties in all worlds.

Thus C becomes the **bridge from existence to necessity**: Because the superposition contains Tr and all worlds emerge from it, Tr holds necessarily in all worlds.

9 Countermodels

The Trinity can be avoided if at least one of the axioms is abandoned:

Abandoned Axiom	Countermodel	Consequence
A1 (Monism)	Dualism (Descartes)	S can exist without T; knowledge and c
A4 (Existence of a world)	Nihilism	There is no world; the entire ontology i
A11 (Transcendental Bridge)	Epistemological Skepticism	Unknowability does not imply fundame
A12 (Experienceability Realization)	Empiricism	Experienceability is not identical with r
ASP1–ASP5 (Superposition)	No superposition	The bridge from existence to necessity

10 Appendix: Complete Axioms and Theorems

10.1 Axioms (Complete)

$$A1 := \Box \neg \exists x \exists y \text{ FundamentalSeparated}(x, y)$$

$$A4 := \Diamond \exists w \text{ World}(w)$$

$$A11 := \forall w (\text{WorldSeparatedFromConsciousness}(w) \leftrightarrow \neg \exists c (\text{Consciousness}(c) \wedge c(w)))$$

$$A12 := \forall p. \text{Experienceable}(p) \leftrightarrow \exists w. \text{Realized}(w, p)$$

$$A13 := \Box \forall x (T(x) \rightarrow U(x))$$

$$A14 := \Box \forall x (U(x) \rightarrow S(x))$$

$$A15 := \Box \forall x (S(x) \rightarrow T(x))$$

$$ASP1 := C@w_{\text{super}}$$

$$ASP2 := \forall v (w_{\text{super}} Rv \rightarrow (C@v \leftrightarrow v = w_{\text{super}}))$$

$$ASP3 := \forall x \text{ Tr}(x)@w_{\text{super}}$$

$$ASP4 := \forall v (w_{\text{super}} Rv \rightarrow \forall x \text{ Tr}(x)@v)$$

$$ASP5 := \forall v (v \neq w_{\text{super}} \rightarrow w_{\text{super}} Rv)$$

10.2 Definitions

$\text{WorldSeparatedFromConsciousness}(w) := \neg \exists c (\text{Consciousness}(c) \wedge c(w))$

$\text{Experienceable}(p) := \exists w. (\text{Consciousness}(w) \wedge \text{Realized}(w, p))$

$T := \{x \mid U < x < S\}$

$U := \lim_{x \rightarrow \inf} T$

$S := \lim_{x \rightarrow \sup} T$

$Tr := U \wedge T \wedge S$

10.3 Derived Theorems

$\exists T$ (from A4)

$\Diamond C$ (A5 – from A1, A11)

$\forall p (\Diamond p \rightarrow \exists w. \text{Realized}(w, p))$ (A2 – from A1, A11, A12)

$\Box(T \rightarrow U)$ (from Case 1)

$\Box(U \rightarrow S)$ (from Case 2 + Superposition)

$\Box(S \rightarrow T)$ (from Case 3)

$\Rightarrow \Box(U \leftrightarrow T) \wedge \Box(T \leftrightarrow S) \wedge \Box(S \leftrightarrow U)$

$\Rightarrow \Box(U \wedge T \wedge S)$

11 Conclusion

This treatise has shown:

1. **In pure S5**, the Trinity is **not provable** (Part I).
2. **In the extended system S5+SP** (with superposition axioms), the Trinity is **provable** (Part II).
3. The crucial metaphysical burden rests on the superposition axioms – they are **not logical theorems**, but **additional assumptions**.

The formal work has thus precisely articulated the logical and metaphysical requirements of such a proof. The crucial question – whether the superposition axioms are true – remains a matter for philosophical or physical investigation. Logic has done its work: it has explicated the consequences of the assumptions and clearly named the limits of pure S5.

The Trinity is not an additional entity, but a structural condition – yet it is provable only under the superposition hypothesis.

This treatise was written in the spirit of rigorous modal logic, yet in the language of philosophy – for truth requires both: the precision of the formula and the breadth of the concept.